An Empirical Equilibrium Model of the Italian Government Debt Market in the Euro area

Carlo A. Favero and Luca Gagliardone*

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Abstract

The equilibrium in the market for the Italian government debt is determined by the interaction of monetary, fiscal and debt management authorities and the markets. Total debt is composed of long-term bonds and short-term bills. In the long-run equilibrium the entire debt is financed by long-term bonds, In the short-run, however, bills are issued as a consequence of (stationary) expectations errors on the price of debt. Non-linear demand and supply of bonds are identified by independent monetary and fiscal policies. The model fits the data very well with a unique equilibrium. The feasibility of debt stabilization in Italy in a regime of QE exit is addressed in an out-of-sample simulation.

JEL Codes: C32, H63

Keywords: empirical equilibrium model, Italian Government Debt

*Favero: Deutsche Bank Chair, IGIER-Bocconi, and CEPR carlo.favero@unibocconi.it, Gagliardone: Bocconi University, luca.gagliardone@unibocconi.it. This paper is part of the ITFIN project at Ministero dell'Economia e Finanze, aimed at building a stock-flow consistent model of the Italian economy in the euro area. We thank, without implicating them, Riccardo Barbieri, Claudio Cicinelli, Francesco Felici, Francesco Nucci, Ottavio Ricchi, Christian Tegami, for their many comments and suggestions.

1 Introduction

This paper develops an empirical equilibrium model of the Italian government debt in the Euro area. The equilibrium is determined by the interaction of fiscal authority, debt management authority, monetary policy authority, households, insurance companies and pension funds, financial companies and the rest of the world. Total debt is made of long-term bonds and short-term bills. In the long-run equilibrium the entire debt is financed by long-term bonds. Given the expectations of the price at which it will be able to sell them, the debt manager issues in each period enough bonds to cover the entire deficit. Once the price of the long-term bonds is revealed and the associated expectation error is observed, the debt manager finances any further borrowing by issuing short-term bills at a given price, mainly determined by the monetary policy authority. This mechanism captures very well the features of the data that show a common trend between total debt and long-term bonds and stationary Trasury bills. The equilibrium in the market for long-term bonds is determined by the match of aggregate demand and supply. The main source of identification of the model is the relevance of monetary policy as a shifter of the aggregate demand for debt, and that of fiscal policy as a shifter of the aggregate supply of debt. Concentrating on data from the Euro period helps identification in that (i) fiscal policy is conducted by local authorities and it reacts to domestic macroeconomic variables while monetary policy is conducted by the European Central Bank and it reacts to global Euro area condition (ii) monetary policy is conducted both via conventional and unconventional measures: while conventional monetary policy has an impact both on the demand and the supply side of the market for government debt, QE affects directly only the demand side (iii) the occurrence of the subprime lending crisis and the Euro area debt has generated remarkable variability in the demand and supply shifters that helps the work of the instruments in tracing more precisely aggregate demand and supply of debt. The supply of government bonds is determined by the debt dynamics, which in each period depends on the the existing stock of debt, the primary surplus and cost of financing the debt. The demand of governments bonds is the aggregation of the demand of five sectors: Households, Financial Companies, Insurance Companies and Pension Funds, The Rest of the World, the Bank of Italy (who acts as an ECB agent during QE in Europe). These different sectors feature different drivers of demand that result in an aggregate demand that is non-linear in the price of government bonds. Demand and supply drivers are modelled via Error Correction specifications that allow for differences in the long-run and a short-run dynamics. The model projects the equilibrium for the government debt market by taking as exogenous real growth, inflation, monetary policy, as described by the Euribor 3 months (conventional monetary policy), the quantity of Italian Government Bonds held by the Central Bank (non-conventional monetary policy), and the yield on the 10-year German Government Bonds.

We first illustrate the different building blocks of the model and their interaction, then the fitted behavioural relationships and their capability of replicating the relevant features of the data is evaluated, finally the model is put at work to address the feasibility of debt stabilization in a regime of QE exit by the ECB over the sample 2017-2020. The fiscal-monetary policy mix projected for this period is particularly challenging. On the one hand, the recovery in Europe and the normalization of growth and inflation in the Euro area after the crisis calls from an exit strategy from the unconventional monetary policy implemented by the ECB via quantitative easing; on the other hand, the Italian Ministry of Economic and Finance has set an important fiscal stabilization target for the same period. We build a scenario for the relevant domestic exogenous variables based on the Update to the Economic and Financial Document 2016 published on the website of the Italian Ministry of Economic and Finance. Similarly we build a scenario for the relevant exogenous international variables using projections for the Euro area monetary policy rates and the German 10-year Bunds yield to maturity. We then simulate paths for the fiscal variables to be compared to the targets reported in the same document from which we take the scenario for the domestic exogenous variables.

Starting from Jovanovic and Rousseau (2001), several papers have analyzed theoretically and empirically the supply, Vayanos and Vila (1999),Greenwood and Vayanos (2014), Krishnamurthy and Vissing-Jorgensen (2011) ,Reinhart and Sack (2000) and the demand,Krishnamurthy and Vissing-Jorgensen (2012), for government debt. This paper considers the case of Italy, in which government debt is risky, to model simultaneously demand, supply and debt management. Our specification of the demand functions by the different sectors in the flow of funds matrix is empirical and it is driven by the observation of the importance of non-linearities in the relation between the excess return of Italian bonds on the risk free asset, German Bund, and the quantity of debt allocated to each sector. The demand for Italian long-term bonds by each sector features a dynamics that allows in the long-term (cointegrating) equilibrium for a non-linear dependence on the spread between yield to maturities of 10-year Italian and German bonds. Non- linearities are modelled through the two Hermite polynomials used in term structure models to capture the slope and the curvature of the yield curve (see, for example, Nelson and Siegel (1987), Diebold and Li (2006), Gürkaynak et al. (2007)). These terms allow for a convex and non-linear hump-shaped relation between the spread and quantity demanded, with the location and the form of the hump and the convexity driven by estimated parameters. In principle our specification of the demand for government debt allows for a backward-bending demand curve that might generate multiple equilibria. Multiple equilibria in the government bond market are widely discussed since the contribution of Calvo (1988), who pointed out that, in a model of rational investors, expectations of future default can generate multiple solutions to the price of a bond. Typically a low-rate equilibrium emerges when market considers the probability of default as low, while an high rate equilibrium emerges when the market considers the probability of default as high. These models have been used to understand the behaviour of government bonds yields during the Euro crisis (Corsetti et al. (2014), De Grauwe and Ji (2012)). Our empirical estimation delivers parameters in the demand curve that do not imply a backward bend capable of generating multiple equilibria. However, an interesting shape emerges in which the demand curve is rather flat for values of the spread below 150 basis points to steepen up remarkably for values of the spread above that threshold. Given this shape of the demand function the cost of fiscal irresponsibility becomes rapidly much higher for high values of the spread.

We model the supply side of the market through the reaction function of monetary, fiscal authorities and the debt manager. Our determination of the short-run equilibrium allows the model to closely fit the observed fluctuations in short-term and long-term debt and matches the observations that in the Euro area period long-term debt is the main driver of the Italian debt dynamics, while short-term debt is rather stable over time. This evidence is remarkably different from the one reported and discussed in Missale and Blanchard (1994), who observe a strong inverse relation between the level of the debt and its maturity at high level of debt. Interestingly, Missale and Blanchard (1994) justify the observed relationship in the data with the idea that the government may need to decrease the maturity of the debt as debt increases to maintain the credibility of its anti-inflationary stance. This argument cannot clearly be applied to the Euro area period where anti-inflationary stance is in the mandate of an independent European monetary authority. The strategic behaviour of the debt manager that uses short-term bills to cover the gap between planned debt and actual debt generated by shocks in the price of long term bonds, is consistent with the consideration by Broner et al. (2013) that high public debt economies borrow short term due to the high risk premium charged by bondholders on long-term debt.

2 The Market for the Italian Gross Public Debt: the Data

The dynamics of Italian nominal government debt in the data is:

$$GBills_t + B_{G,t} + L_t = GBills_{t-1} + B_{G,t-1} + L_{t-1} + Def_t + Sfa_t \qquad (1)$$

$GBills_t$ is short-term debt, $B_{G,t}$, is long-term debt, L_t are loans, Sfa_t is a stock-flow adjustment term. The need for stock-flow adjustment arises for example in the presence of revenue from sales or purchases of financial and non-financial assets; revaluations, in the case the debt is valued at market prices; debt write-offs, etc. all items which do not enter the definition of the primary surplus (see Eurostat (2014)).

We consider the following simplified version of the debt dynamics:

$$\begin{aligned} GBills_t + B_{G,t} &= GBills_{t-1} + B_{G,t-1} + Def_t + Res_t \\ Res_t &= Sfa_t - \Delta L_t \end{aligned} \qquad (2)$$

Figure (1) reports the dynamics of the different components of the stock of debt.

INSERT FIGURE (1) HERE.

Figure (1) illustrates that in the Euro area period long-term debt is the trending component of the Italian debt dynamics, while short-term debt is rather stable over time. As a consequence, the share of short term debt decreases for about ten per cent of the total debt at the beginning of the sample to about five per cent of the total debt at the end of the sample. Note also that the term we have labelled as Res_t indeed behaves as a residual, fluctuating around a zero mean, and shows a remarkable correlation with the change in short-term government debt.

Long-term debt $B_{G,t}$ is a allocated to several sectors, namely the central bank ($B^{cb}_{G,t}$), households ($B^{hh}_{G,t}$), insurance companies and pension funds ($B^{icpf}_{G,t}$), financial companies ($B^{fc}_{G,t}$), and the rest of the world ($B^{rw}_{G,t}$):

$$B_{G,t} = B^{cb}_{G,t} + B^{hh}_{G,t} + B^{icpf}_{G,t} + B^{fc}_{G,t} + B^{rw}_{G,t} \qquad (3)$$

Figure (2) reports the time-series behaviour of the debt held by different sectors.

INSERT FIGURE (2) HERE.

Figure (2) illustrates that the single most relevant sector is the rest of the world and that the demand of long-term debt by this sector fluctuated remarkably during the crisis to contract from 0.7 trillions in 2010 to 0.5 trillions at the peak of the crisis in 2012. Interestingly, the demand of debt by domestic financial institutions mimics that of the rest of the world before and after the Euro crises but not during it. While foreign investors were "flying to quality" out of the Italian debt, domestic banks and financial institutions did increase their exposure. The demand of households is peculiar in that it shows a remarkable speed of mean reversion toward a constant. Finally, the quantity allocated to the Bank of Italy, reflects the ECB quantitative easing as most of the purchases of Italian Government Securities by the ECB have been going through the domestic central bank. A better understanding of the behaviour of these different components of the demand of government debt is gauged by inspecting the cross-plots of the different components of the demand for long-term debt and an obvious indicator of risk-premium, the 10-year BTP-BUND spread, reported in Figure (3) for the period 1999-2016.

INSERT FIGURE (3) HERE.

Figure (3.1) plots the 10-year BTP-BUND spread to the quantity of Italian Government Bonds held by the rest of the world. The scatter chart shows a non linear relationship with demand going up with prices at low level of the spread to bend backward at level of the spread around 200 bp when demand becomes inversely related to prices for values of the spread between 200 and 300 basis. For values of the spread higher than 300 hundred basis points demand form the rest of the world does not appear to respond to the spread. Notice that this pattern of non-linearity has the potential of generating multiple equilibria in the debt market. As a matter of fact, the existence of multiple equilibria depend on how the aggregate demand reflects the feature of the rest of the world demand and the evidence that a portion of the demand curve of debt by the rest of the world is backward bending does not necessarily imply the existence of multiple equilibria. Indeed, the strong-pattern of non-linearity is peculiar to the rest of the world as the banking sector the insurance companies and pension funds sector and Households show different patterns of relationship. Figure (3.2) plots the 10-year BTP-BUND spread to the quantity of Italian Government Bonds held by the Households. The scatter chart shows very little sensitivity of the quantity held to the price. Figures (3.3) and (3.4) plots the 10-year BTP-BUND spread to the quantity of Italian Government Bonds held by Financial Companies and the ICPF sector respectively. Here, the pattern in the two figures is very similar with a scatter chart showing a substantially downward sloping demand in prices for both sectors.

3 An Equilibrium Model

Our equilibrium model is constructed by adopting a specification of the dynamics of debt drivers that it used by the debt manager to determine the real supply of government bonds. First the full deficit, given price expectations at time t-1, is entirely financed by long-term government bonds. The equilibrium price of long-term government bond is then determined by equating the supply to the aggregate demand that comes from the different sectors. Once the price is determined short-term debt is used to close the gap generated by expectations errors (i.e. discrepancies between the price expected by the debt manager when setting the supply of long-term bonds and the price that effectively cleared the market) and by the residual term in the debt dynamics.

3.1 The Drivers of the Debt Dynamics

The main driver of the debt dynamics is the total government deficits that we write as follows:

$$Def_t = D_t + I_t - \Pi_t^{CB} \tag{4}$$

where $I_t = i_{A,t} \left(B_{G,t-1} + GBills_{t-1} \right)$ is the nominal financing cost of the total debt and $i_{A,t}$ is the average financing rate for the debt.

D_t is the primary deficit, determined by the difference between non-interest government expenditures and government revenues, $i_{A,t}$ is the average cost of financing the debt, finally Π_t^{cb} are profits remitted by the central bank. This is a small component of the total deficit which is not controlled by the fiscal authority, and it is projected by a simple Error Correction Mechanism in which the long-run remittted profits are proportional to the quantity of long-term bonds held by the central bank:

$$\Delta\Pi_t^{cb} = \alpha_{\Pi,q} + \alpha_{\Pi,1} \left(\Pi_{t-1} - c_{\Pi,2} B_{G,t-1}^{cb} \right) + u_{\Pi,t} \tag{5}$$

where $\alpha_{\Pi,q}$ is the set of quarterly dummies capturing the seasonal component.

The average cost of financing the debt,$i_{A,t}$, is projected as

$$\Delta i_{A,t} = \alpha_{A,q} + \alpha_{A,1}\Big(i_{A,t-1} - i_{A,t-4}\Big) + \alpha_{A,2}\Big(i_{A,t-1} - c_{A,2} r_{t-1}^{10Y} - c_{A,3} r_{t-1}^{12M}\Big) + u_{A,t} \quad (6)$$

where, as before, $\alpha_{A,q}$ is the set of quarterly dummies.[1]

The yield to maturity on long term debt is determined endogenously by the model, given the level of the yield to maturity on German 10-year government bonds and aggregate demand and supply of Italian government bonds.

The yield on short-term debt is instead anchored to the monetary policy rate (Euribor 3 months) r_t^E in the long-run, but in the short-run, during episodes of "flight to quality", it is allowed to react to changes in the risk-premium, as captured by the BTP-BUND spread:

$$\Delta r_t^{12M} = \alpha_{r,0} + \alpha_{r,1} \Delta r_t^E + \alpha_{r,2} \Delta\Big(r_t^{10Y} - r_t^{10Y,Ger}\Big) + \alpha_{r,3}\Big(r_{t-1}^{12M} - r_{t-1}^E\Big) + u_{r,t} \quad (7)$$

A fiscal reaction function determines the ratio of primary surplus to GDP $d_t = \frac{D_t}{Y_t}$:

$$\Delta d_t = \alpha_{d,q} + \alpha_{d,1}\Big(d_{t-1} - c_{d,2} g_{t-1}^r - c_{d,3} i_{A,t} b_{t-1}\Big) + u_{P,t} \quad (8)$$

The primary surplus dynamically reacts to GDP driven by an output stabilization motive, captured by the response to real GDP growth g^R, and a debt stabilization motive, captured by the response to the total cost of financing the debt to GDP ratio $\frac{I_t}{Y_t} = i_{A,t} b_{t-1}$. This fiscal rule it is not derived by any optimization and it is in line with the heuristic approach proposed by Taylor (1993), Taylor (2000) to model the behaviour of monetary and fiscal authorities[2].

3.2 The Demand for Long Term Debt

The aggregate demand for long-term debt, $B_{G,t}$, is a composition of the individual demands of several sectors, namely the central bank ($B_{G,t}^{cb}$ exogenous), households ($B_{G,t}^{hh}$), insurance companies and pension funds ($B_{G,t}^{icpf}$), financial companies ($B_{G,t}^{fc}$), and the rest of the world ($B_{G,t}^{rw}$):

$$B_{G,t} = B_{G,t}^{cb} + B_{G,t}^{hh} + B_{G,t}^{icpf} + B_{G,t}^{fc} + B_{G,t}^{rw} \quad (9)$$

[1]In the simulations presented, the financing cost calculated using the average financing rate $i_{A,t}$ is smoothed over the four-quarters period using a moving average.

[2]Symmetrically to the financing cost, the primary deficit and the deficit presented in the simulations are smoothed over the year using an annual moving average.

Consistently with the preliminary evidence from the data illustrated in the second section we propose a general specification for the demand of bond in sector i capable of accommodating persistence and non linearities:

$$\begin{aligned}\Delta \log\left(B^{i}_{G,t}\right) = {} & \beta_{0,i} + \beta_{1,i}\log\left(B^{i}_{G,t-1}\right) + \beta_{2,i}y_{t-1} + \beta_{3,i}\pi_{t-2} + \\ & + \beta_{4,i}\left(\frac{1-\frac{\exp\left(-\left(r^{10Y}_{t-1}-r^{10Y,Ger}_{t-1}\right)\right)}{\tau_1}}{\frac{\left(r^{10Y}_{t-1}-r^{10Y,Ger}_{t-1}\right)}{\tau_1}} - \frac{\exp\left(-\left(r^{10Y}_{t-1}-r^{10Y,Ger}_{t-1}\right)\right)}{\tau_1}\right) + \\ & + \beta_{5,i}\left(\frac{1-\frac{\exp\left(-\left(r^{10Y}_{t-1}-r^{10Y,Ger}_{t-1}\right)\right)}{\tau_2}}{\frac{\left(r^{10Y}_{t-1}-r^{10Y,Ger}_{t-1}\right)}{\tau_2}}\right) + u_{i,t}\end{aligned} \tag{10}$$

where π_t si the annual inflation and y_t is the annual log GDP.

The demand for Italian long-term bonds by each sector features a dynamics that allows in the long-term (cointegrating) equilibrium for a non-linear dependence on the spread between yield to maturities of 10-year Italian and German bonds. The non linearities are modelled through the two Hermite polynomials used in term structure models to capture the slope and the curvature of the yield curve (see, for example, Nelson and Siegel (1987), Diebold and Li (2006), Gürkaynak et al. (2007)). These terms allows for a convex and a non-linear hump-shaped relation between the spread and quantity demanded, with the location and the form of the hump and the convexity driven by estimated parameters.

For each sector we adopt a version of this general specification with a parameterization nested in the linear model.

The demand of Italian bonds by the rest of the world reflects the prominence of the non-linearity in the spread:

$$\begin{aligned}\Delta \log\left(B^{rw}_{G,t}\right) = {} & \alpha_{rw,0} + \alpha_{rw,1}\Bigg(\log\left(B^{rw}_{G,t-1}\right) - c_{rw,2}y_{t-1} + \\ & - c_{rw,3}\left(\frac{1-\frac{\exp\left(-\left(r^{10Y}_{t-1}-r^{10Y,Ger}_{t-1}\right)\right)}{\tau_{rw}}}{\frac{\left(r^{10Y}_{t-1}-r^{10Y,Ger}_{t-1}\right)}{\tau_{rw}}} - \frac{\exp\left(-\left(r^{10Y}_{t-1}-r^{10Y,Ger}_{t-1}\right)\right)}{\tau_{rw}}\right)\Bigg) + u_{1,t}\end{aligned} \tag{11}$$

In addition the demand of the rest of the world is scaled by the (lagged) level of GDP.

The demand of insurance companies and pension funds as well as financial companies are modelled as Error Correction Models that incorporate two long-run relationships: the first one, between the government bonds held by financial companies, the spread between the 10 year yield on Italian and German government bonds, and the scale factor of the (lagged) level of GDP; the second one among the government bonds held by financial companies and by the ICPF sector. The spread enters the equation non-linearly, allowing for an upper limit to the quantity that the two sectors are willing to buy for increasingly higher spread values.

$$\Delta \log\left(B_{G,t}^{fc}\right) = \alpha_{fc,0} + u_{1,t} + \\ + \alpha_{fc,1}\left(\log\left(B_{G,t-1}^{fc}\right) - c_{fc,2}y_{t-1} - c_{fc,3}\left(\frac{1-\frac{\exp\left(-\left(r_{t-1}^{10Y}-r_{t-1}^{10Y,Ger}\right)\right)}{\tau_{fc}}}{\frac{\left(r_{t-1}^{10Y}-r_{t-1}^{10Y,Ger}\right)}{\tau_{fc}}}\right)\right) \tag{12}$$

$$\Delta \log(B_{G,t}^{icpf}) = \alpha_{icpf,0} + \alpha_{icpf,1}\Big(\log(B_{G,t-1}^{icpf}) - c_{icpf,2}\log(B_{G,t-1}^{fc})\Big) + u_{3,t} \tag{13}$$

Consistently with the very small observed flutctuations, the households' demand is modelled through an ECM, with an high estimated spped of adjustment towards equilibrium, in which the only long-run time varying driver is the annual inflation rate:

$$\Delta \log(B_{G,t}^{hh}) = \alpha_{hh,0} + \alpha_{hh,1}\Big(\log(B_{G,t-1}^{hh}) - c_{hh,2}\pi_{t-2}\Big) + u_{4,t} \tag{14}$$

3.3 Long-run Equilibrium in the Government Debt Market

The long-run equilibrium dynamic system of demand and supply described above depends on its steady-state solution, which in turn is determined by the long-run debt demand and the long-run debt supply.

The long-run debt demand is obtained by aggregating the equilibrium demands of each sector in our model, including the exogenous demand of public debt by the Central Bank, which are specified as follows:

$$\bar{B}^{rw}_G = \exp\left(\hat{c}_{rw,1} + \hat{c}_{rw,2}\bar{y} + \hat{c}_{rw,3}\left(\frac{1-\frac{\exp\left(-\left(\bar{r}^{10Y}-\bar{r}^{10Y,Ger}\right)\right)}{\hat{\tau}_{rw}}}{\frac{\left(\bar{r}^{10Y}-\bar{r}^{10Y,Ger}\right)}{\hat{\tau}_{rw}}} - \frac{\exp\left(-\left(\bar{r}^{10Y}-\bar{r}^{10Y,Ger}\right)\right)}{\hat{\tau}_{rw}}\right)\right)$$

$$\bar{B}^{fc}_G = \exp\left(\hat{c}_{fc,1} + \hat{c}_{fc,2}\bar{y} + \hat{c}_{fc,3}\left(\frac{1-\frac{\exp\left(-\left(\bar{r}^{10Y}-\bar{r}^{10Y,Ger}\right)\right)}{\hat{\tau}_{fc}}}{\frac{\left(\bar{r}^{10Y}-\bar{r}^{10Y,Ger}\right)}{\hat{\tau}_{fc}}}\right)\right)$$

$$\bar{B}^{icpf}_G = \exp\left(\hat{c}_{icpf,1} + \hat{c}_{icpf,2}\log(\bar{B}^{fc}_G)\right)$$

$$\bar{B}^{hh}_G = \exp\left(\hat{c}_{hh,1} + \hat{c}_{hh,2}\bar{\pi}\right)$$

$$\bar{B}_G = B^{cb}_G + \bar{B}^{hh}_G + \bar{B}^{icpf}_G + \bar{B}^{fc}_G + \bar{B}^{rw}_G$$

where $\hat{c}_{i,1}$ is the long-run estimates for intercept in sector i.

INSERT FIGURE (4) HERE.

Similarly the long-run debt supply is obtained by considering the equilibrium solution for the debt supply side of our model.

Given the debt to GDP dynamics :

$$\Delta b_t = \frac{i_{A,t} - g^n}{1+g^n} b_{t-1} + d_t - \pi^{CB}_t \tag{15}$$

where $g^r = g^n - \pi$ is the equilibrium real growth rate (difference between the nominal equilibrium growth rate g^n and the equilibrium inflation π), we can obtain the long-run solution[3]:

$$\bar{b}^s = \frac{1+g^n}{g^n - \bar{\imath}_A\left(1+\hat{c}_{d,3}\right)}\left(\hat{c}_{d,1} + \hat{c}_{d,2}g^r - \bar{\pi}^{CB}\right) \tag{16}$$

By rescaling $\bar{B}_G$ by the level of GDP we obtain $\bar{b}^d$ and we can analyse the long-run equilibrium properties of the model.

Fiscal policy is the main driver of the supply of debt and the relation between debt and the spread depends crucially on the reaction function of the fiscal authority. In particular, in analogy to the famous Taylor principle

[3]The derivation of the formula is in the Appendix. This simplified version of the long-run debt supply does not take into account the quarterly timing of the estimated equations, but is valid for annual data: the correction of the formula that is used to obtain the figures and is specific for the estimated equations is presented in the Appendix.

Taylor (1993), the long-term supply is negatively sloped in the space spanned by the spread and the debt to GDP ratio only if the fiscal authority reacts with sufficient intensity to the fluctuations in the cost of financing the debt (i.e. if $\hat{c}_{d,3}$ is smaller than -1). Shifts in fiscal policy move the debt supply schedule and identify the long-run demand for debt. Conventional monetary policy (as measured by $\bar{r}^E$) affects both the demand and supply of debt and it is not a strong source of identification, while unconventional monetary policy, as measured by the public debt held by the central bank only affects equilibrium demand for debt and it is a source of identification for supply.

Interestingly by specifying equilibrium values[4] for the exogenous variables, i.e. GDP growth ($g^r = 0.5\%$), inflation ($\pi = 2\%$), the yield to maturity on the 10-year German Bond $\left(\bar{r}^{10Y,Ger} = 3\%\right)$, the monetary policy rate ($\bar{r}^E = 1.5\%$), and the quantity of debt held by the central bank B_G^{cb} equal to roughly 12.5% of Italian GDP in 2016), we can plot the equilibrium demand and supply.

INSERT FIGURE (5) HERE.

INSERT FIGURE (6) HERE.

In principle, our specification of the demand for government debt, which includes a non-linear portion coming from the rest of the world and the financial companies, allows for a backward bending demand curve that might generate multiple equilibria. Multiple equilibria in the government bond market are widely discussed since the contribution of Calvo (1988), who pointed out that, in a model of rational investors, expectations of future default can generate multiple solutions to the price of a bond. Typically a low-rate equilibrium emerges when market considers the probability of default as low, while an high rate equilibrium emerges when the market consider the probability of default as high. These models have been used to understand the behaviour of government bonds yields during the Euro crisis (Corsetti et al. (2014), De Grauwe and Ji (2012)). Our empirical estimation delivers parameters in the demand curve that do not imply a backward bend capable of generating multiple equilibria. However, an interesting shape emerges in which the demand curve is rather flat for values of the spread below 150 basis points to steepen up remarkably for values of the spread above that threshold. Given this shape of the demand function the cost of fiscal irresponsibility becomes rapidly much higher for high values of the spread. Different variability of the spread at high and low level of this variable does not necessarily require the existence of multiple equilibria.

[4]Average values for the period 2001-2016.

3.4 Equilibrium in the government bond market

The short-run equilibrium in the government debt market is different form the long-run one because of the presence of short-run dynamics in all the variables and of shocks to the economy. Our mechanism for the determination of such equilibrium hinges on the behaviour of the debt manager that takes for given monetary and fiscal policy and interacts with the market. Our preliminary graphical analysis of the data show that the dynamics of the stock of long-term bonds mimics closely that of the debt while short term bills have a behaviour over time that is consistent with that of a shock-absorber. Consistently with this evidence we posit that the real supply of government bonds is determined by the debt manager so that long-term bonds are used to finance the full deficit given a conjecture for bond prices based on information available at time t-1:

$$b_{G,t} = \frac{B_{G,t-1}}{E\left(P_{G,t}\right)} + \frac{Def_t}{E\left(P_{G,t}\right)} \tag{17}$$

The debt manager expectations are adaptive:

$$E\Big(P_{G,t}\Big) = P_{G,t-1} + \kappa\left[E\Big(P_{G,t-1}\Big) - P_{G,t-1}\right]$$

where κ is set to 0.1 in our empirical analysis.

The price of long-term debt is then set by equating real supply to nominal demand

$$P_{B,t} = \frac{B_{G,t}}{b_{G,t}} \tag{18}$$

the price of long-term government bonds determines the yield to maturity

$$\log(1 + r_t^{10Y}) = \beta_{0,r} + \beta_{1,r}\log\left(P_{B,t}\right) + u_{p,t} \tag{19}$$

where $\beta_{1,r}$ can be interpreted as the inverse of the duration and the intercept term and $u_{p,t}$ capture measurement errors.

Once the price of bonds is determined, Bills are then used to cover the gap generated by expectations errors (and the stock-flow adjustment) :

$$\begin{aligned} \Delta GBills_t &= E\left(P_{G,t}\right)\left(b_{G,t} - \frac{B_{G,t-1}}{E\left(P_{G,t}\right)}\right) - \left(B_{G,t} - B_{G,t-1}\right) + Res_t \\ &= \Big(E\left(P_{G,t}\right) - P_{G,t}\Big)b_{G,t} + Res_t \end{aligned} \tag{20}$$

This implies that when there is a spike in the cost of financing long-term debt and the expected equilibrium interest price on debt is higher than the actual, then actual debt needed is higher than expected at the emission of long-term debt and the gap is filled using short-term debt.

INSERT FIGURE (7) HERE.

It is easy to check that this mechanism of the determination of the supply of bills delivers:

$$GBills_t + B_{G,t} = B_{G,t-1} + GBills_{t-1} + Def_t + Res_t. \tag{21}$$

4 Model Evaluation

The model is estimated over a sample of quarterly data for the Euro period 2000Q1-2016Q4. Model evaluation is based on assessing the significance of the estimated coefficients and the dynamic behaviour of endogenous variables in the several Error Correction Specifications and by simulating the model dynamically within sample over 2010Q1-2016Q4. All estimated equations, reported in an Appendix 2, feature precisely estimated coefficients and there are no cases of ill-behaving residuals. In the within-sample dynamic simulation, the model projects the (partial) equilibrium for the government debt market by taking as exogenous real growth, inflation, monetary policy, as described by the Euribor 3 months (conventional monetary policy) and the quantity of Italian Government Bonds held by the Central Bank (non-conventional monetary policy), and the yield on the 10-year German Government Bonds[5]. The results are illustrated in Figure (8), where we report simulated values with their 95 per cent confidence interval plotted along with the observed values for the main endogenous variable of the model, the spread on German Bunds, the yield to maturity on bills, the debt ot GDP ratio, the (annual) deficit to GDP ratio, the (annual) primary deficit to GDP ratio, the (annual) cost of financing the debt as a percentage of GDP, the nominal bond demand and its components. Observed values for virtually all variables are within the simulated confidence bounds with the exception of the peak of the Euro crisis period, in which both the spikes in the spread and in the yield to maturity on GBills and the response of fiscal policy to the crisis are not captured by our fitted equations. Notice that, in absence of the extraordinary fiscal policy response to the crisis, the debt to GDP ratio is projected at a higher level

[5]In addition, we take also as exogenous some residual items, such as the stock-flow adjustments in the debt dynamics and the quantity of loans by the Government sector. In the out-of-sample simulations, these items are naturally set to be constant at their average values in the estimation period.

than the observed one.

INSERT FIGURE (8) HERE.

4.1 The Feasibility of Public Debt Stabilization in scenarios of QE exit

The period 2017-2020 offers an interesting opportunity to put the model at work in an out-of-sample simulation. The fiscal-monetary policy mix projected for this period is particularly challenging. On the one hand, the recovery in Europe and the normalization of growth and inflation in the Euro area after the crisis calls from a gradual exit from the unconventional monetary policy implemented by the ECB via quantitative easing; on the other hand, the Italian Ministry of Economic and Finance has set an important fiscal stabilization target for the same period. We build a scenario for the relevant domestic exogenous variables based on the Update to the Economic and Financial Document 2016 published on the website of the Italian Ministry of Economic and Finance. Similarly we build a scenario for the relevant exogenous international variables using projections for the Euro area monetary policy rates and the German 10-year Bunds yield to maturity. We then use the model to simulate paths for the fiscal variables to be compared to the targets reported in the same document from which we take the scenario for the domestic exogenous variables.

Table (1) illustrates the set-up for our simulations by reporting the path for the exogenous domestic and international variables and the target path for the endogenous fiscal variables.

	Source	Variable	2017	2018	2019	2020
Domestic Scenario	MEF[6]	GDP[7]	1.5	1.2	1.2	1.3
	MEF	Inflation[8]	0.6	1.8	1.8	1.7
International Scenario	ECB[9]	Euribor 3M	-0.3	-0.3	-0.1	0.4
	DB[10]	10-year Bund	0.3	0.5	0.8	1.0
Targets[11]	MEF[12]	Deficit	2.1	1.6	0.9	0.2
	MEF	Primary deficit	-1.7	-2.0	-2.6	-3.3
	MEF	Financing cost	3.8	3.6	3.5	3.5
	MEF	Gross Debt	131.6	130.0	127.1	123.9

Table 1: Scenarios and targets

To evaluate the impact of monetary and fiscal policy in determining the equilibrium in the debt market we simulate the model out-of-sample for the period 2017-2020 by considering two alternative monetary and fiscal policy scenarios.

We consider two alternative QE exit scenarios: a gradual exit in which bond buying by the CB is stopped but the stock is held constant, and a more aggressive exit scenario in which bonds held by CB are progressively reduced to their level at pre non-conventional monetary policy period. We pair the simulation of the two alternative QE scenarios with two alternative fiscal policy scenarios: a ”normal” fiscal policy scenario and an ”aggressive” fiscal policy scenario. In the ”normal” scenario the fiscal authority follows the estimated fiscal reaction function with an equilibrium response of the (quarterly) primary surplus to (annual) output growth and an equilibrium response of the (quarterly) primary surplus to the (quarterly) cost of financing debt over GDP at the point estimates, which are respectively 0.25 and 1.21. In the ”aggressive” scenario we increase the equilibrium response of the (quarterly)

[6] Updates DEF (Economic and Financial Document). Ministry of Economic Affairs. September 2017. Table I.2, page 7.

[7] Percentage variations.

[8] GDP deflator percentage variations.

[9] ECB staff macroeconomic projections for the Euro area. European Central Bank. March 2018. Page 4.

[10] Outlook for the German economy, monthly report. Deutsche Bundesbank. December 2017. Page 7.

[11] As a percentage of GDP.

[12] Updates DEF (Economic and Financial Document). Ministry of Economic Affairs. September 2017. Table I.4, page 11.

primary surplus to (annual) output growth and an equilibrium response of the (quarterly) primary surplus to the (quarterly) cost of financing debt over GDP by one standard deviation, respectively to 0.35 and 1.55. In the spirit of Leeper and Zha (2003) we consider these variations as a modest policy intervention that does not significantly shift agents' beliefs about policy regime and does not induce the changes in behavior that Lucas Jr (1976) emphasizes. Figure (9)-(10) reports the outcome of simulations with the two alternative monetary policy scenarios paired first with the "normal" fiscal policy and second with the "aggressive" fiscal policy. To facilitate legibility of the figures only 95 percent confidence intervals associated to the more aggressive monetary policy scenario are reported.

INSERT FIGURE (9) HERE.
INSERT FIGURE (10) HERE.

Figure (9) clearly illustrates that under the normal fiscal policy scenario debt stabilization is out of reach independently from the monetary policy scenario. Given the "normal" fiscal policy, the debt to GDP ratio is projected to be rather stable, around 1.28, but definitely higher than the target of 1.23 over the period 2017- 2020 independently from the adopted exit strategy by the ECB. Interestingly, the exit strategy makes a difference for the market of long-term government debt, with a much higher spread and a strong reallocation of quantity held from the central bank and the rest of the world to domestic financial companies, insurance companies, and pension funds. This asymmetry does create important consequences in the debt dynamics as it is only partially reflected in the cost of financing the debt that responds slowly to fluctuations in the spread. Fluctuations in the spread do not affect the cost of financing the stock of long-term debt but only that of new issues. A higher primary surplus in the case of aggressive monetary policy is not sufficient to compensate for the higher cost of financing the debt and the deficit to GDP ratio is permanently at an higher level in the case of aggressive monetary policy, although there is no important divergence in the paths of deficit to GDP between the two alternative monetary policy scenarios.

Figure (10) strengthens the message in Figure (9) illustrating that under the "aggressive" fiscal policy scenario debt stabilization is achieved independently from the monetary policy scenario. The level of the debt projected for 2020 is indeed very close to the target value for both alternative exit strategies, although it is marginally lower in the case of the aggressive exit strategy. This outcome depends on the stronger reaction in the fiscal reaction function to the increased cost in financing the debt caused by the effect on the level of long-term yields of the exit strategy.

Overall the results of model simulation strongly indicate that the feasibility of the fiscal target depends much more on the behaviour of the domestic fiscal policy authority than on that of the European monetary authority.

5 Conclusions

We have estimated and simulted an empirical equilibrium model for the Italian government debt. The behaviour of the debt manager in the model delivers a long run equilibrium different from the short-run equilibrium. In the long-run the the entire debt is financed long-term, while short-term debt is used to cover gaps generated by wrong conjectures on bond prices by the debt manager. This mechanism allows a close track of the trends observed in the data.
Independent fluctuations in monetary and fiscal policy allow to pin down precisely the demand and supply of debt. Error correction models are successfull in describing the dynamics of the data along both curves. The specification of the demand for government debt, allows in principle for a backward-bending demand curve, determined by the behaviour of the foreign sector, that might generate multiple equilibria. Our empirical estimation delivers parameters in the demand curve that do not imply a backward bend capable of generating multiple equilibria. However, an interesting shape emerges in which the demand curve is rather flat for values of the spread below 150 basis points to steepen up remarkably for values of the spread above that threshold. Given this shape of the demand function, the cost of fiscal irresponsibility becomes rapidly much higher for high values of the spread.

The period 2017-2020 offers an interesting opportunity to put our model at work. We addressed the question of the feasibility of fiscal stabilization in a regime of QE exit in out-of-sample simulation. Our results indicate feasibility of the fiscal stabilization target of a debt to GDP ratio of 123 in 2020. The possibility of achieving such a target depends much more on the behaviour of the domestic fiscal policy authority than on that of the European monetary authority.

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5.1 Appendix: Derivation of long-run equilibrium

Denote the annual interest rate between $t-4$ and $t-1$ with $i_A = i^A_{t-4,t-1}$, the annual growth rate (nominal and real) with $g = g_{t-3,t}$, and the annual GDP $y = y_{t-3,t} = y_t + y_{t-1} + y_{t-2} + y_{t-3}$. Consider the debt dynamics:

$$\begin{aligned} B_{G,t} =& B_{G,t-4}(1+i_A) + PrDef_{t-3} + PrDef_{t-2} + PrDef_{t-1} + PrDef_t \\ b_{G,t} =& b_{G,t-4}\frac{(1+i_A)}{(1+g^n)} + d_{t-3}\frac{y_{t-3}}{y} + d_{t-2}\frac{y_{t-2}}{y} + d_{t-1}\frac{y_{t-1}}{y} + d_t\frac{y_t}{y} \end{aligned}$$

where $g^r = g^n - \pi$ is the equilibrium annual real growth rate, which is the difference between the annual nominal equilibrium growth rate g^n and the equilibrium annual inflation π.

In the long-run, the quarterly fiscal reaction function implies that:

$$\begin{aligned} d_{t-3} &= \hat{c}_{d,1} + \hat{c}_{d,2}g^r + \hat{c}_{d,3}i^A_{t-4,t-3}\frac{B_{G,t-4}}{y_{t-3}} \\ d_{t-2} &= \hat{c}_{d,1} + \hat{c}_{d,2}g^r + \hat{c}_{d,3}i^A_{t-3,t-2}\frac{B_{G,t-3}}{y_{t-2}} \\ d_{t-1} &= \hat{c}_{d,1} + \hat{c}_{d,2}g^r + \hat{c}_{d,3}i^A_{t-2,t-1}\frac{B_{G,t-2}}{y_{t-1}} \\ d_t &= \hat{c}_{d,1} + \hat{c}_{d,2}g^r + \hat{c}_{d,3}i^A_{t-1,t}\frac{B_{G,t-1}}{y_t} \end{aligned}$$

The long-run quarterly value for $i^A_{t-i,t-i+1}$ is obtained from equation (6):

$$i^A_{t-i,t-i+1} = \hat{c}_{A,1} + \hat{c}_{A,2}\left(\left(r^{10Y}_{t-i} - r^{10Y,Ger}_{t-i}\right) + r^{10Y,Ger}_{t-i}\right) + \hat{c}_{A,3}r^{12M}_{t-i}$$

and the long-run quarterly rate on GBills is equal to the equilibrium monetary policy rate, r^E:

$$r^{12M}_{t-i} = r^E_{t-i}$$

The long-run equilibrium supply can be found by imposing $b_t = b_{t-4} = \bar{b}^s$ and by substituting for long-term equilibria relationships determining the deficit to GDP ratio, d_t,. In the steady state we assume that annual and quarterly variables are linked through $g^n_{t-i,t-i+1} = \frac{1}{4}g^n$ and $i^A_{t-i,t-i+1} = \frac{1}{4}i_A$ for every i and t.

Thanks to the relation between quarterly and annual growth, we can rewrite the debt/GDP ratio in the fiscal reaction function as:

$$\frac{B_{G,t-4}}{y_{t-4,t-7}}\frac{y_{t-4,t-7}}{y_{t-3}} = b_{G,t-4}\frac{y_{t-7}\Big(1+(1+\frac{1}{4}g^n)+(1+\frac{2}{4}g^n)+(1+\frac{3}{4}g^n)\Big)}{y_{t-7}\Big(1+g^n\Big)} = b_{G,t-4}\frac{4+\frac{3}{2}g^n}{1+g^n}$$

for every t, and using the fact that at the equilibrium $b_{t-i} = b_t$ for every i and t, then:

$$d_{t-3} = d_{t-2} = d_{t-1} = d_t = \hat{c}_{d,1} + \hat{c}_{d,2}g^r + \hat{c}_{d,3}\frac{i_A}{4}b_{G,t-4}\frac{4+\frac{3}{2}g^n}{1+g^n}$$

Now notice that:

$$\frac{y_{t-3}}{y} = \frac{y_{t-3}}{y_{t-3}+y_{t-2}+y_{t-1}+y_t} = \frac{1}{1+(1+\frac{g^n}{4})+(1+2\frac{g^n}{4})+(1+3\frac{g^n}{4})}$$

$$\frac{y_{t-2}}{y} = \frac{(1+\frac{g^n}{4})}{1+(1+\frac{g^n}{4})+(1+2\frac{g^n}{4})+(1+3\frac{g^n}{4})}$$

$$\frac{y_{t-1}}{y} = \frac{(1+2\frac{g^n}{4})}{1+(1+\frac{g^n}{4})+(1+2\frac{g^n}{4})+(1+3\frac{g^n}{4})}$$

$$\frac{y_t}{y} = \frac{(1+3\frac{g^n}{4})}{1+(1+\frac{g^n}{4})+(1+2\frac{g^n}{4})+(1+3\frac{g^n}{4})}$$

therefore by substitution:

$$\begin{aligned} b_{G,t} &= b_{G,t-4}\frac{(1+i_A)}{(1+g^n)} + \Big(\hat{c}_{d,1} + \hat{c}_{d,2}g^r + \hat{c}_{d,3}\frac{i_A}{4}b_{G,t-4}\frac{4+\frac{3}{2}g^n}{1+g^n}\Big) \\ &= b_{G,t-4}\left[\frac{(1+i_A)}{(1+g^n)} + \hat{c}_{d,3}i_A\frac{1+\frac{3}{8}g^n}{(1+g^n)}\right] + \Big(\hat{c}_{d,1}+\hat{c}_{d,2}g^r\Big) \end{aligned}$$

Subtract $b_{G,t-4}$ on both sides:

$$0 = b_{G,t-4}\left[\frac{(i_A - g^n) + \hat{c}_{d,3}i_A\Big(1+\frac{3}{8}g^n\Big)}{(1+g^n)}\right] + \Big(\hat{c}_{d,1}+\hat{c}_{d,2}g^r\Big)$$

Dropping time subscripts, the supply of debt is:

$$\bar{b}^s = -\Big(\hat{c}_{d,1}+\hat{c}_{d,2}g^r\Big)\frac{(1+g^n)}{(\bar{i}_A - g^n) + \hat{c}_{d,3}\bar{i}_A\Big(1+\frac{3}{8}g^n\Big)}$$

The only difference with respect to the annualized formula is the additional scale factor $1+\frac{3}{8}g^n$, which accounts for the quarterly growth of the GDP.

5.2 Appendix: The Empirical Model

In this section the results of the estimation are presented, together with the R^2, the Durbin-Watson statistics, the mean of the dependent variable, and the standard errors for the estimated equations; standard errors are in parentheses.

5.2.1 The Drivers of the Debt Dynamics

Equation (5):

$$\Delta\Pi_t^{cb} = \underset{(61.04)}{51.60}(q=1) + \underset{(48.18)}{256.67}(q=2) + \underset{(61.26)}{67.26}(q=3) + \underset{(50.31)}{251.63}(q=4) + \\ - \underset{(0.105)}{0.59}\left(\Pi_{t-1} - \underset{(0.0005)}{0.0075}B_{G,t-1}^{cb}\right) + \hat{u}_{\Pi,t}$$

R^2: 0.783, DW: 2.462, Mean: 21.23, S.E. of the regression: 94.91.

Equation (6):

$$\Delta i_{A,t} = \underset{(0.0007)}{0.003}(q=1) + \underset{(0.0006)}{0.004}(q=2) + \underset{(0.0007)}{0.003}(q=3) + \underset{(0.0006)}{0.004}(q=4) + \\ - \underset{(0.067)}{0.387}\left(i_{A,t-1} - i_{A,t-4}\right) - \underset{(0.078)}{0.604}\left(i_{A,t-1} - 0.25 \times \underset{(0.071)}{0.108}r_{t-1}^{10Y} - 0.25 \times \underset{(0.068)}{0.688}r_{t-1}^{12M}\right) + \hat{u}_{A,t}$$

R^2: 0.894, DW: 1.945, Mean: -0.0001, S.E. of the regression: 0.0006.

Equation (7):

$$\Delta r_t^{12M} = \underset{(0.0005)}{0.0002} + \underset{(0.128)}{0.959}\Delta r_t^E + \underset{(0.134)}{0.963}\Delta\left(r_t^{10Y} - r_t^{10Y,Ger}\right) - \underset{(0.062)}{0.162}\left(r_{t-1}^{12M} - r_{t-1}^E\right) + \hat{u}_{r,t}$$

R^2: 0.662, DW: 2.245, Mean: -0.0008, S.E. of the regression: 0.003.

Equation (8):

$$\Delta d_t = \underset{(0.011)}{0.061}(q=1) - \underset{(0.012)}{0.009}(q=2) + \underset{(0.011)}{0.024}(q=3) + \underset{(0.011)}{0.015}(q=4) + \\ - \underset{(0.111)}{0.601}\left(d_{t-1} + \underset{(0.095)}{0.258}g_{t-1}^r + \underset{(0.368)}{1.231}i_{A,t}b_{t-1}\right) + \hat{u}_{P,t}$$

R^2: 0.935, DW: 2.068, Mean: 0.0002, S.E. of the regression: 0.009.

5.2.2 The Demand for Long Term Debt

Equation(11):

$$\Delta \log\left(B_{G,t}^{rw}\right) = \underset{(1.787)}{-3.243} - \underset{(0.127)}{0.310}\left(\log\left(B_{G,t-1}^{rw}\right) - \underset{(0.294)}{1.653} y_{t-1} + \right.$$

$$\left. - \underset{(0.328)}{1.026}\left(\frac{1 - \frac{\exp\left(-\left(r_{t-1}^{10Y} - r_{t-1}^{10Y,Ger}\right)\right)}{\underset{(0.086)}{0.403}}}{\frac{\left(r_{t-1}^{10Y} - r_{t-1}^{10Y,Ger}\right)}{\underset{(0.086)}{0.403}}} - \frac{\exp\left(-\left(r_{t-1}^{10Y} - r_{t-1}^{10Y,Ger}\right)\right)}{\underset{(0.086)}{0.403}}\right)\right) + \hat{u}_{1,t}$$

R^2: 0.115, DW: 1.659, Mean: 0.006, S.E. of the regression: 0.042.

Equation (12):

$$\Delta \log\left(B_{G,t}^{fc}\right) = \underset{(1.848)}{-0.055} + \hat{u}_{2,t}$$

$$- \underset{(0.029)}{0.103}\left(\log\left(B_{G,t-1}^{fc}\right) - \underset{(1.251)}{0.919} y_{t-1} - \underset{(0.4555)}{2.076}\left(\frac{1 - \frac{\exp\left(-\left(r_{t-1}^{10Y} - r_{t-1}^{10Y,Ger}\right)\right)}{\underset{(0.368)}{0.552}}}{\frac{\left(r_{t-1}^{10Y} - r_{t-1}^{10Y,Ger}\right)}{\underset{(0.368)}{0.552}}}\right)\right)$$

R^2: 0.240, DW: 2.624, Mean: 0.015, S.E. of the regression: 0.061.

Equation (13):

$$\Delta \log(B_{G,t}^{icpf}) = \underset{(0.381)}{0.708} - \underset{(0.056)}{0.170}\Big(\log(B_{G,t-1}^{icpf}) - \underset{(0.099)}{0.669}\log(B_{G,t-1}^{fc})\Big) + \hat{u}_{3,t}$$

R^2: 0.167, DW: 2.416, Mean: 0.010, S.E. of the regression: 0.055.

Equation (14):

$$\Delta \log(B_{G,t}^{hh}) = \underset{(2.984)}{0.784} - \underset{(0.066)}{0.257}\Big(\log(B_{G,t-1}^{hh}) - \underset{(23.125)}{5.627}\,\pi_{t-2}\Big) + \hat{u}_{4,t}$$

R^2: 0.245, DW: 1.826, Mean: −0.002, S.E. of the regression: 0.098.

5.2.3 Equilibrium in the government bond market

Equation (19):

$$\log(1+r_t^{10Y}) = \underset{(0.00002)}{0.0004} - \underset{(0.000)}{0.097} \log\left(P_{B,t}\right) + \hat{u}_{p,t}$$

R^2: 0.999, DW: 0.461, Mean: 0.040, S.E. of the regression: 0.000.

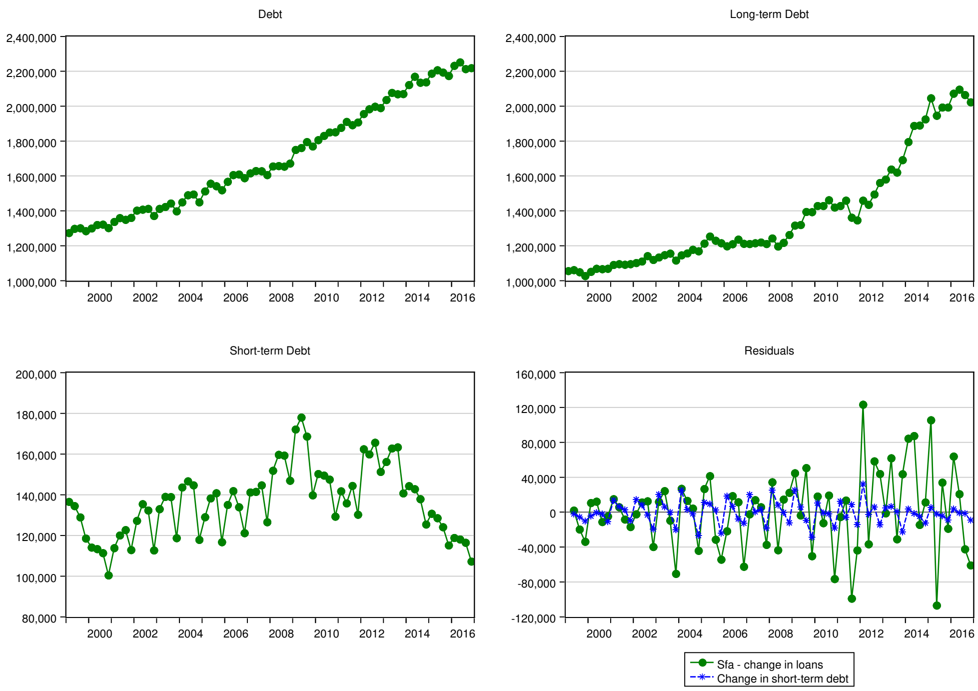


Figure 1: The Debt Dynamics

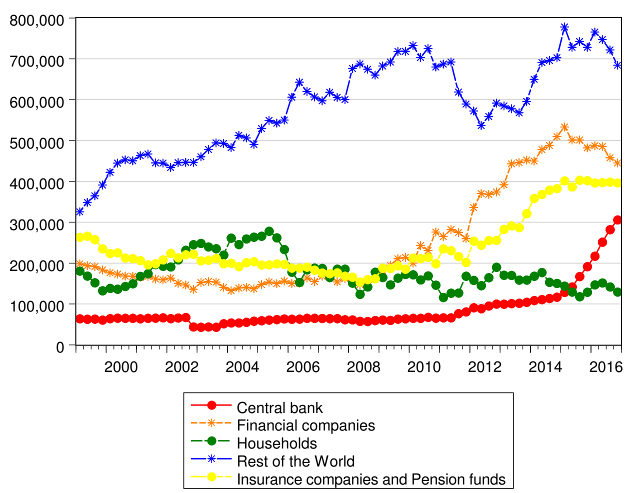


Figure 2: The structure of the demand of long term debt

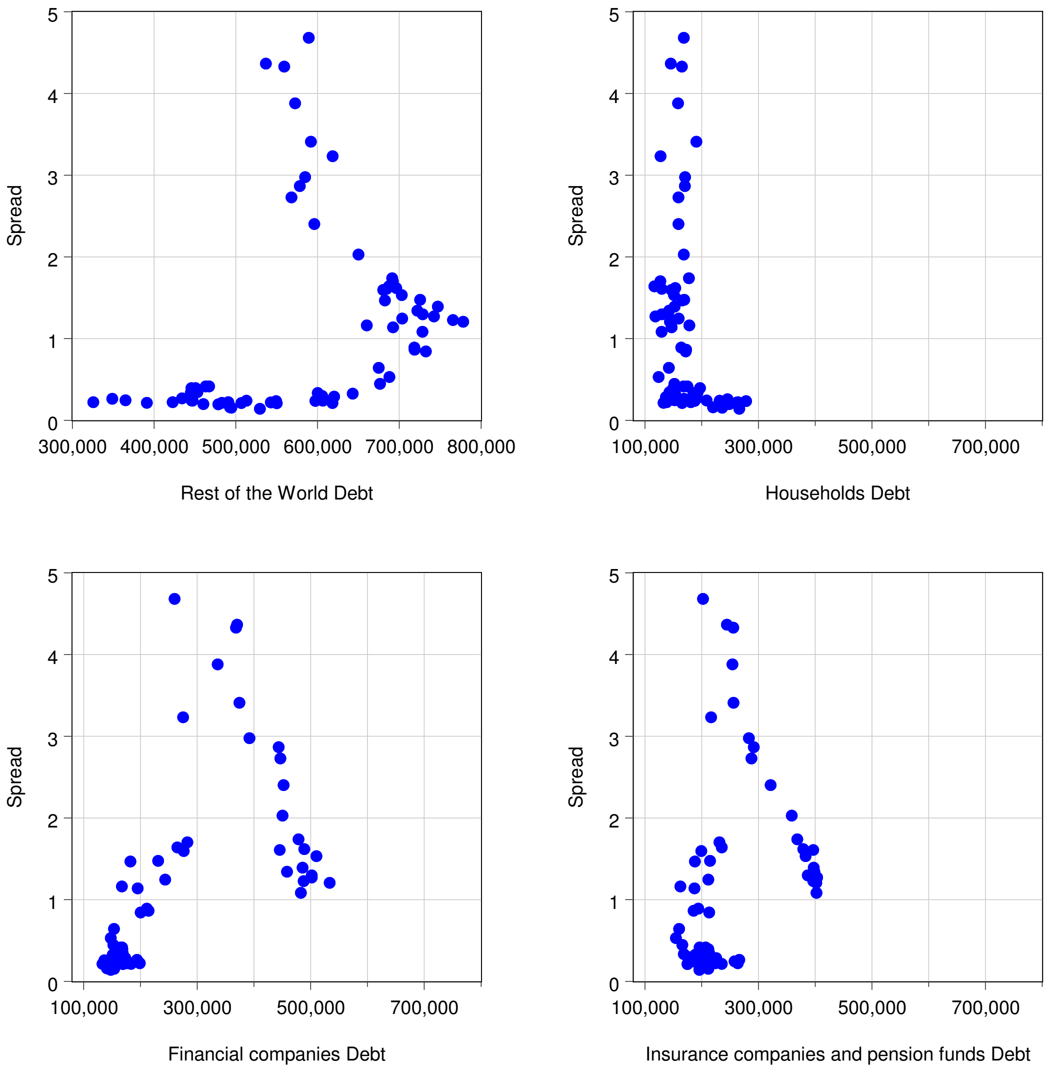


Figure 3: The Components of the Demand

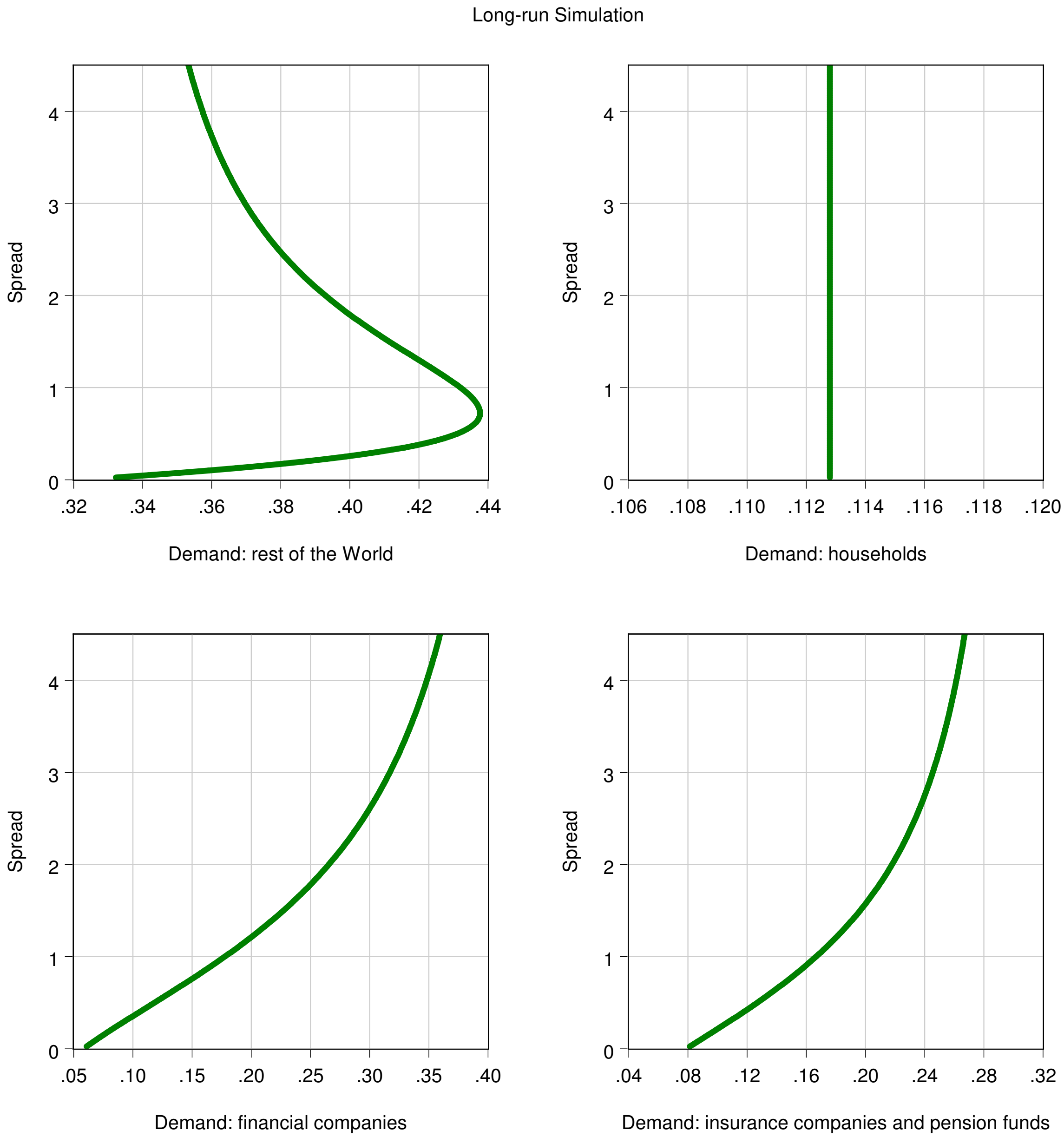
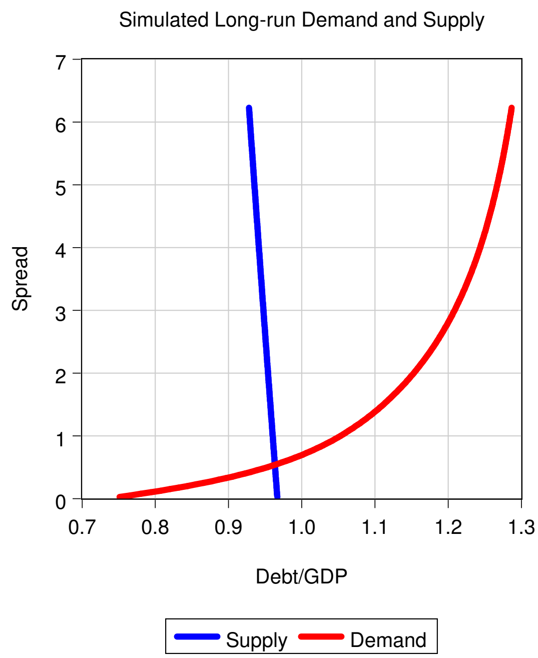


Figure 4: Components of the long-run Demand



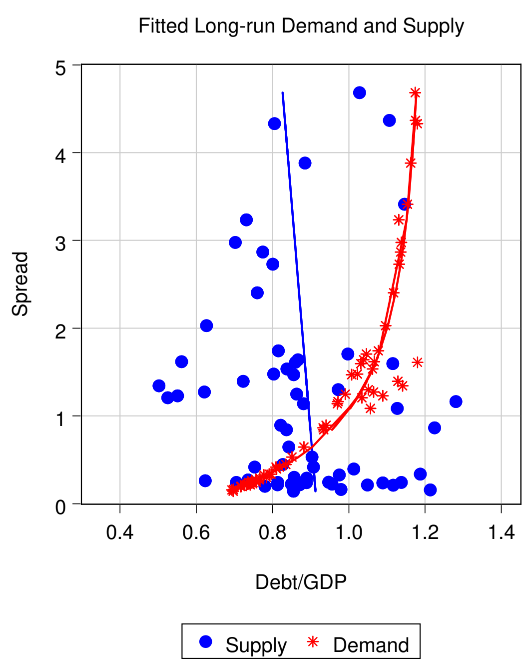


Figure 5: The long-run Equilibrium in the Government Debt Market

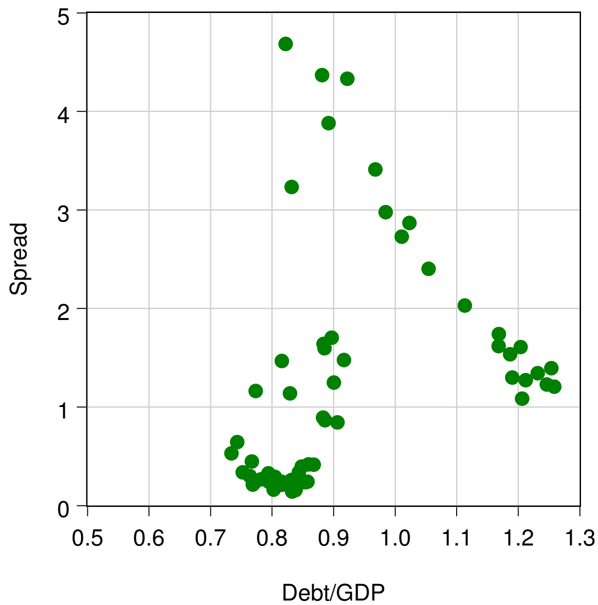


Figure 6: Cross-plot of Debt/GDP and 10Y BTP-BUND spread

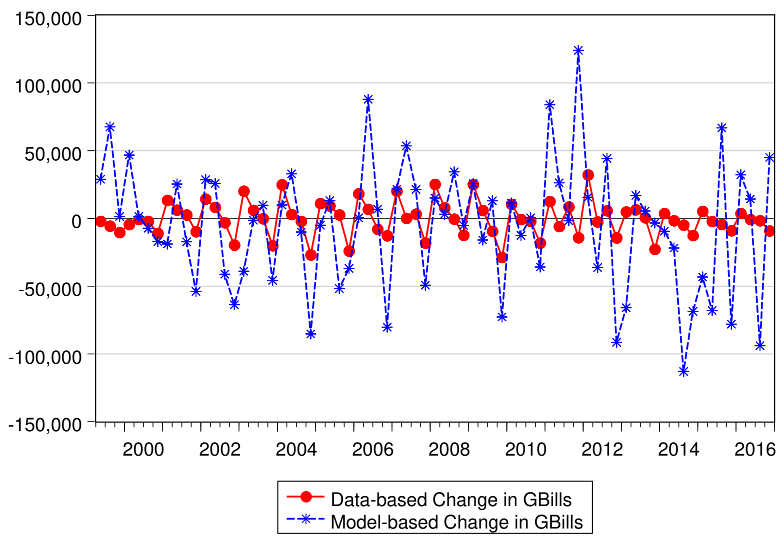


Figure 7: Model-based vs data-based GBills quantity

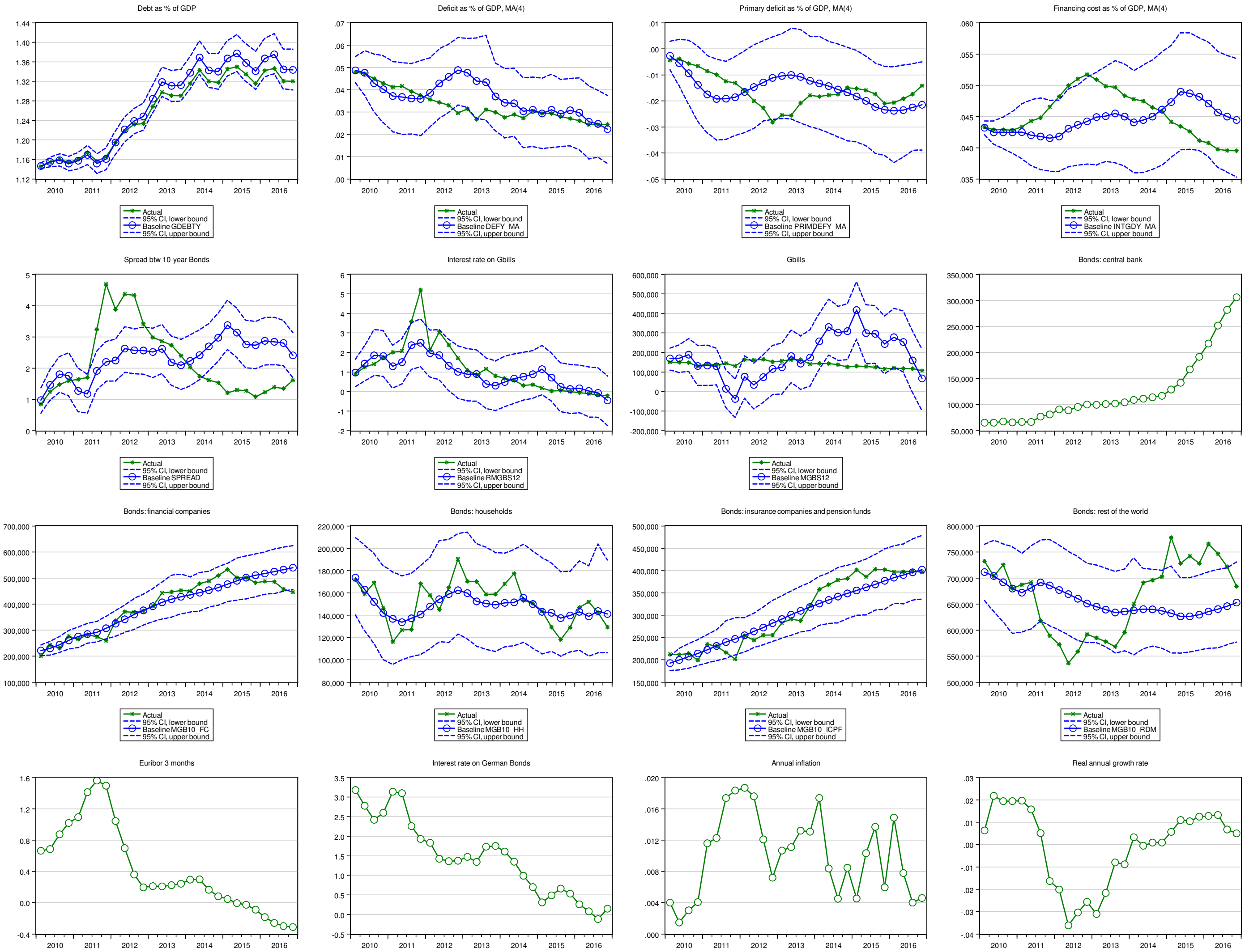


Figure 8: within sample dynamic simulation. 2010-2016

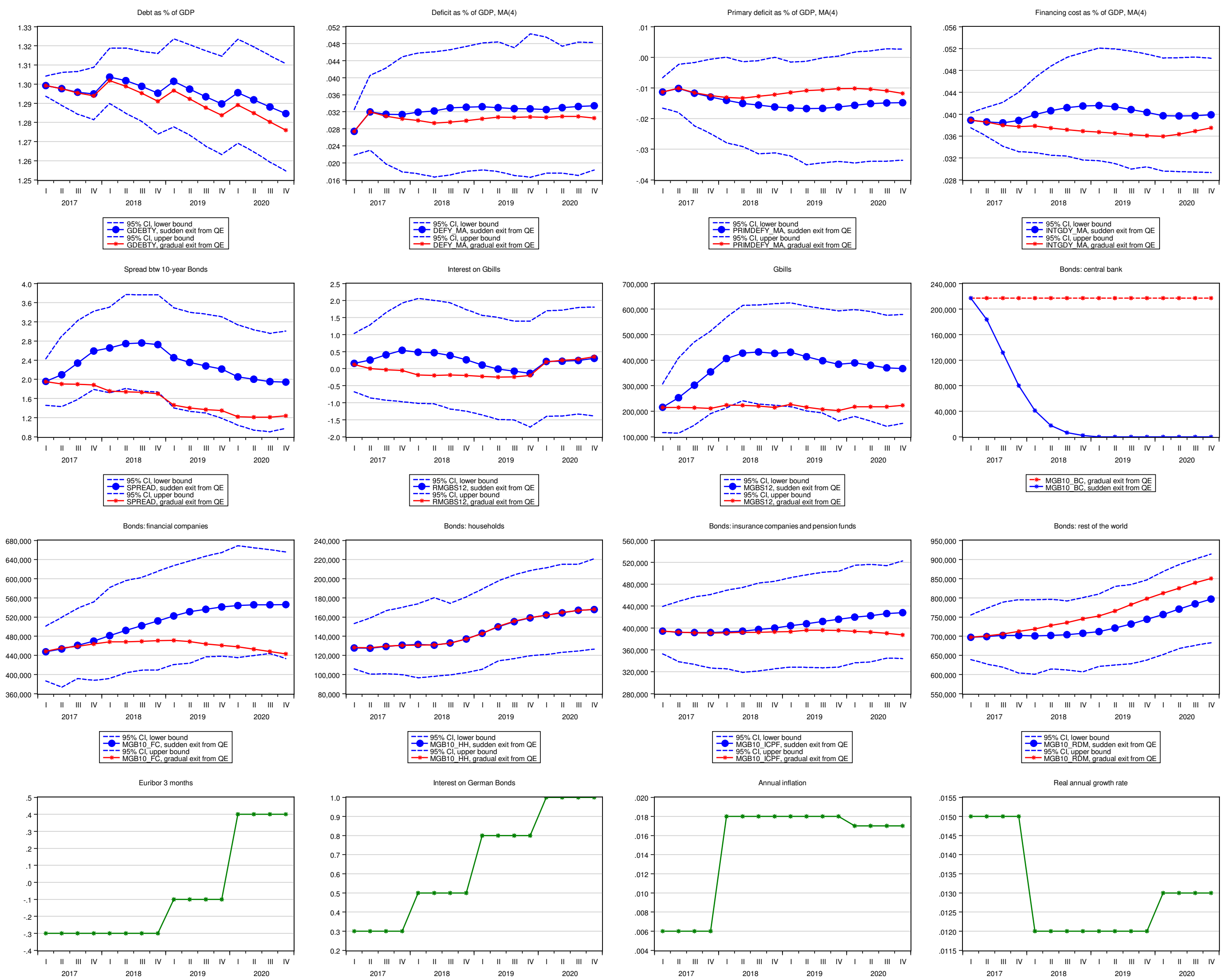


Figure 9: Out-of-sample simulation. "Normal" fiscal policy scenario

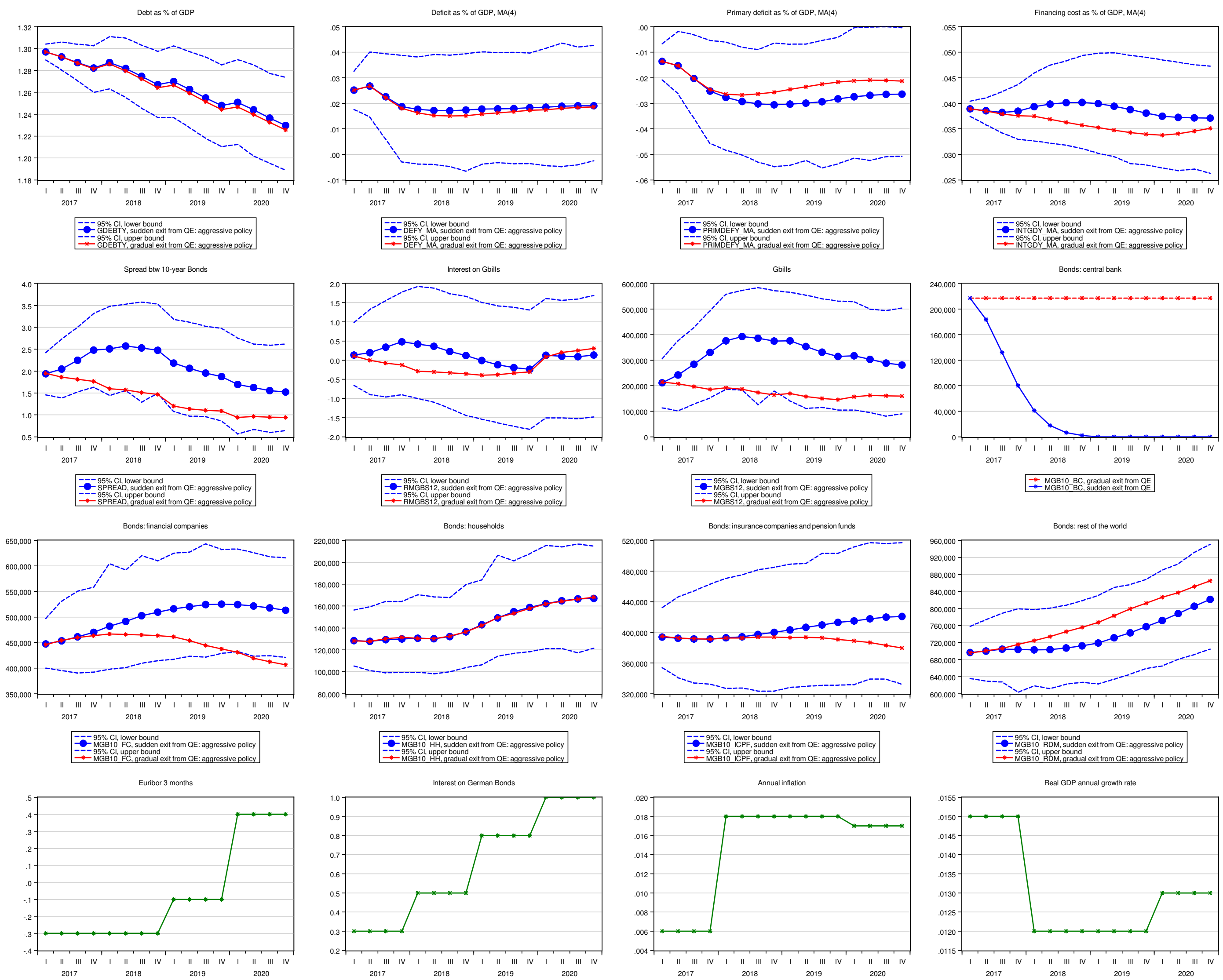


Figure 10: Out-of-sample simulation."Aggressive" fiscal policy scenario

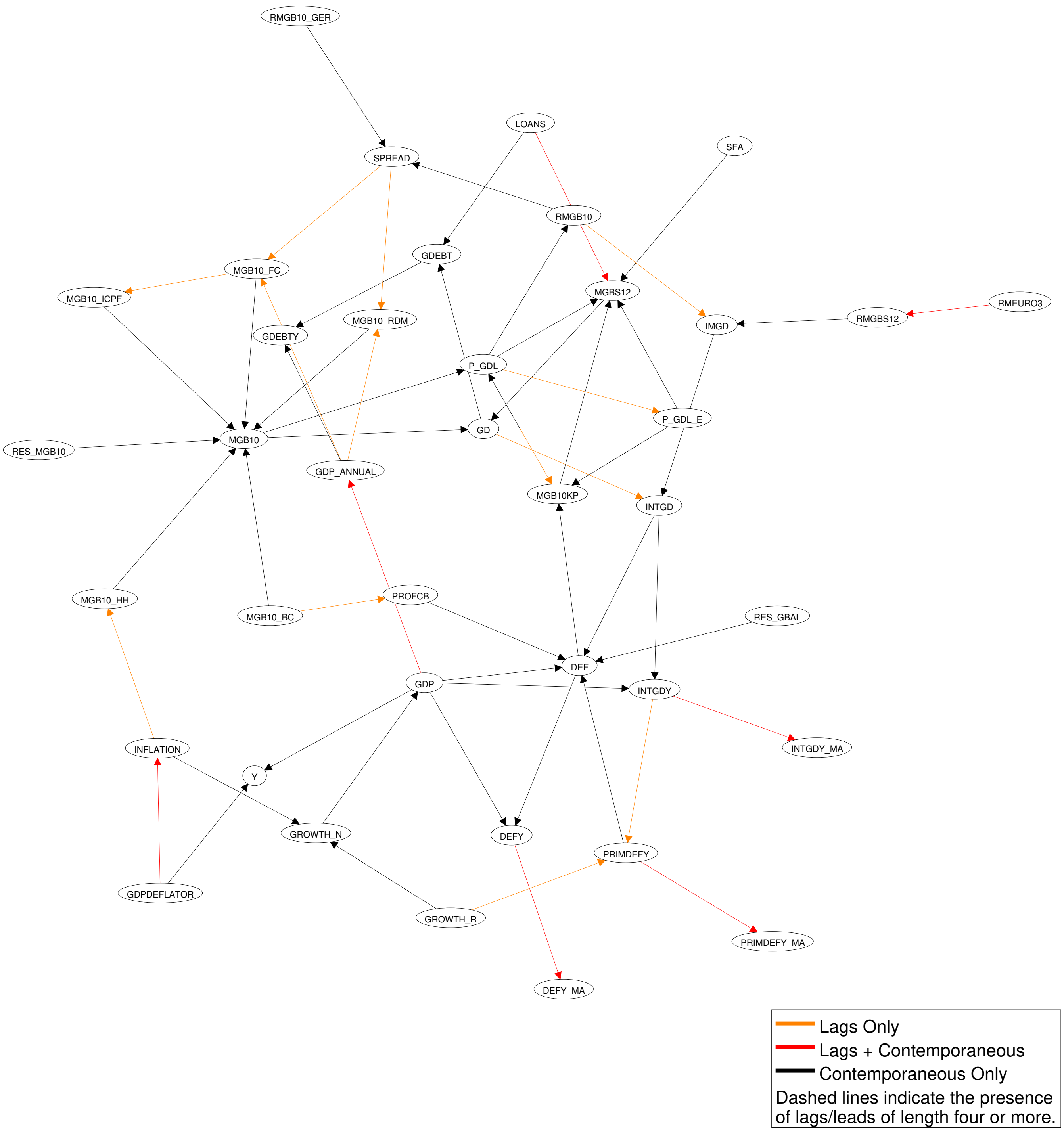


Figure 11: The dynamic dependence graph of the model

Legend of Figure (11)			
Exogenous	growth_r	: Real (annual) growth	
	gdpdeflator	: GDP deflator (base 2009)	
	loans	: Loans, deposits and stock options	Banca d'Italia
	sfa	: Stock-flow adjustments	
	mgb10_bc	: Bond demand of the central bank	Banca d'Italia
	rmeuro3	: Euribor 3 months	Eurostat
	rmgb10_ger	: Yield to maturity on 10-year Bunds	Eurostat
	res_gbal	: Residuals on deficit	Banca d'Italia
	res_mgb10	: Residuals on bond demand	Banca d'Italia
Endogenous	gdp	: Nominal gdp	ISTAT
	y	: Real GDP	Eurostat
	growth_n	: Nominal GDP growth	
	def	: deficit	ISTAT
	defy	: Deficit/GDP	
	primdef	: Primary deficit	ISTAT
	primdefy	: Primary deficit/GDP	
	gd	: Debt (not including loans)	ISTAT
	gdebt	: Debt (including loans)	ISTAT
	debty	: Debt/GDP	
	imgd	: Financing rate on debt	
	intgd	: Average Financing cost on debt	ISTAT
	intgdy	: Financing cost/GDP	
	mgb10	: Bond demand	Banca d'Italia
	mgb10_fc	: Bond demand of financial companies	Banca d'Italia
	mgb10_hh	: Bond demand of households	Banca d'Italia
	mgb10_icpf	: Bond demand of ICPF	Banca d'Italia
	mgb10_rdm	: Bond demand of rest of the world	Banca d'Italia
	mgb10kp	: Real bond supply	
	mgbs12	: GBills	Banca d'Italia
	p_gdl	: Price Bonds	Datastream
	p_gdl_e	: Expectation on price	
	profcb	: Central bank's profits	Banca d'Italia
	rmgb10	: Yield to maturity Bonds	Banca d'Italia
	rmgbs12	: Yield to maturity GBills	Banca d'Italia
	spread	: Spread BTP-Bunds	Datastream