Optimal Public Communication

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Abstract

A benevolent planner chooses optimally whether and how to disclose publicly a private forecast of fundamentals to a large number of informed small agents. These agents interact in economic environments with information frictions, strategic complementarity or substitutability in actions, and a rich set of externalities that are responsible for inefficient fundamental and non-fundamental fluctuations. First, I characterize the optimal policy as a function of the externalities of the economy, the quality of the forecast of the planner, and agents' prior uncertainty. Next, I discuss and interpret the theoretical results within the context of an application to central bank communication.

Keywords: Bayesian persuasion, information design, central bank communication, beauty contest, incomplete information, strategic uncertainty.

1 Introduction

Many economic environments feature a large number of small agents that take decisions under fundamental uncertainty over an unknown state. In general equilibrium, any information available to the agents is used not only to forecast the state, but also to predict other players' actions, as those decisions have an impact on aggregate quantities and prices. It is well known from the work of Morris and Shin (2002) that, whereas private information (such as signals extracted from market interactions) leads in general to better decisions taken by economic agents, public information (such as public announcements by institutions) can sometimes have negative welfare effects. The intuition for this result follows from the fact that public information fosters coordination among strategic economic agents, which may be undesirable because it leads to non-fundamental volatility and sentiment-driven fluctuations, or may be desirable because it reduces the cross-sectional dispersion of actions. For example, in the application to central bank communication that will be discussed later, the former volatility is that of the output gap, the latter dispersion is that of prices.

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In this article, I investigate how public communication can be designed optimally to maximize welfare in these environments. Public communication is here modelled as a public disclosure rule of the imperfect private information that a benevolent planner has about fundamentals. The optimal design of a public communication entails characterizing when committing to such a rule is ex-ante beneficial for welfare and, in particular, how to disclose that information in the best possible way. This can lead to simple forms of communication, such as a commitment to not disclose any information or fully reveal all the available knowledge, or strategic forms of communication, such as partial disclosure of the available knowledge aimed at influencing agents' beliefs in a specific direction.

To abstract from institutional details and identify general principles while retaining tractability, the first part of the paper approaches the question in the class of quadratic economies studied in Angeletos and Pavan (2007), that has been used by previous literature to discuss welfare effects of information. This framework allows for either strategic complementarity or substitutability in actions that capture salient forces of general equilibria, as well as for a rich set of externalities that are responsible for possible wedges between the equilibrium and the efficient use of information, and thus create a scope for policy interventions. Whereas previous work has analyzed comparative statics of welfare with respect to the precision of a gaussian public signal, this paper uses and extends tools[1] from the information design literature to characterize the optimal public communication policy for any possible information structure. This characterization leads to novel insights about the social value and optimal use of public information, as explained in due course below.

The theoretical results of the first part of the paper are then interpreted through the lenses of a business-cycle model with pricing complementarities and nominal rigidities arising from information frictions. In this environment, I solve the problem of a central bank committing to an optimal communication strategy, in order to maximize welfare and stabilize inflation and output gap. I consider two sources of fluctuations in the economy, productivity and markup shocks, to highlight how the optimal communication changes depending on whether fundamental fluctuations are efficient or not. The optimal communication also depends on the level of prior uncertainty of firms in the economy, and the quality or credibility of the information available to the central bank. As a rule of thumb, high uncertainty of firms and high quality of information increase the incentives towards disclosure, whereas inefficient fluctuations from countercyclical markup shocks reduce those incentives. Therefore, the optimal communication can entail full, partial, or no disclosure of the information available to the central bank, depending on which of the above forces is mostly relevant in terms of welfare losses.

The Abstract Setting. The model is a quadratic game with imperfect information. There is a large number of ex-ante identical small agents that take

[1] The algebra is hardly tractable but everything can be solved in closed form. The main steps are reported in Appendix C. Code for checking the detailed steps is provided.

a continuous action. Quadratic utility depends on one's own action, the mean and variance of actions in the population, and a binary fundamental. Individual actions can be strategic complements or substitutes. Information is imperfect as agents only observe a private and (potentially) a public signal informative about the unknown fundamental. Taking as given information and aggregates, agents maximize their expected utility.

The only policy instrument considered in the setting is a communication policy. A benevolent planner observes a noisy private signal informative about the fundamental, and has the option to reveal in full or in part that signal. Extending the work of Kamenica and Gentzkow (2011) to allow for the possibility that the sender does not observe directly the state, the policy is modelled as a mapping from the realization of planner's private signal to a distribution over the public signal, and it is chosen with commitment to maximize welfare before the fundamental is drawn according to its distribution and signals are sent.

In the absence of externalities, the optimal communication always fully reveals planner's private signal. Following the literature, two types of distortions are therefore introduced. First, the economy can be inefficient, which means that, even in the absence of an information friction, actions taken by agents do not maximize welfare. This occurs as agents are small and do not internalize the effect of their decisions on aggregates. An example of an efficient economy is a business-cycle model with productivity shocks, an example of an inefficient one is a model with markup shocks. Had the planner access to state-contingent fiscal policy, it would be optimal to use it to counteract markup-driven fluctuations, but not productivity-driven fluctuations. In the absence of such a fiscal instrument, communication can sometimes be used to reduce the inefficiency. The second type of distortion is a coordination externality, which can arise only when information is incomplete and actions are complements or substitutes. The externality is present when coordination among agents is inefficiently high or low. For example, when firms compete à la Cournot, coordination is inefficiently low, which means that profits would be higher if firms were to perceive stronger strategic complementarities. An example of inefficiently high level of coordination is competition à la Bertrand.

Optimal Use of Public Information. The main theoretical results of the paper characterize how the optimal public communication changes depending on the distortions present in the economy, the prior level of uncertainty of the agents in the economy, and the precision of the information (or credibility) available to to the planner. In particular, the precision of the information available to the planner implies an additional constraint on the choice of the optimal policy that limits the power of communication. This "information-aggregation constraint" captures the imperfect ability of the planner to collect information that is dispersed, and difficult or costly to aggregate.

In efficient economies, precise information is always fully revealed, even in the presence of a coordination externality. Imprecise information, on the other hand, can be harmful when the level of coordination is inefficient, as it may contradict with agents' prior belief causing an increase in uncertainty, rather than a reduction of it. This may lead to non-disclosure being optimal, or a strategic partial revelation being optimal. In inefficient economies, when less information generates efficiency gains, there is an additional incentive towards non-disclosure. The starkest comparison with efficient economies is for the case in which the planner can directly observe the state, which would imply always full revelation in efficient economies. For a small inefficiency, full revelation remains optimal. For a large enough inefficiency, not disclosure is optimal. When inefficiency and coordination externality are of the same magnitude in terms of welfare losses, there is a meaningful tradeoff between the two, that results in partial revelation to be optimal for some levels of prior uncertainty.

Illustration: Central Bank Communication. The theoretical results are interpreted in the context of a microfounded business cycle model with nominal rigidities arising from incomplete information and pricing complementarities similar to Hellwig (2005). Firms compete monopolistically and produce with a technology that can accommodate for increasing or decreasing returns to scale. Pricing decisions are taken on the basis of imperfect knowledge about the realization of a aggregate productivity or markup shock, whereas labor adjusts flexibly to clear markets. Also the central bank is subject to an information friction, i.e. only observes an imperfect private signal that is informative about the realization of the shock. Before shocks realize and signals are sent, the central bank commits to a communication policy, that is chosen optimally to maximize expected utility of the representative household.

The coordination externality here implies that inflation is inefficiently volatile, and thus the central bank has an incentive to let the output gap fluctuate more to stabilize inflation. When fluctuations are driven by productivity, and the central bank has access to precise information about the realization of the shocks, there is no tradeoff between stabilization of inflation and the output gap, and the optimal policy achieves approximately the first best. A tradeoff between the two arises when private knowledge of the monetary authority is imprecise, and results in a condition on preferences, technology, and information for inflation targeting to be optimal. When this condition is met and firms hold little prior uncertainty about the realization of the productivity shock, a non-disclosing policy is optimal. On the other hand, if firms are highly uncertain about the realization of the shock, partial or full revelation are optimal. When fluctuations are driven by a countercyclical markup shock, the central bank has an additional incentive towards non-disclosure of information, because of the efficiency gains that this generates. In particular, the optimal communication turns out to be non-disclosing when firms have little prior uncertainty about the realization of the shock, whereas it can be partially revealing or fully revealing when firms are highly uncertain. The latter case relies upon a condition on preferences, technology, and information for markup targeting to be optimal.

2 The Abstract Setting

The abstract setting is a quadratic game with imperfect information and strategic complementarities as in Angeletos and Pavan (2007). The state is binary, which is an essential assumption for the analysis to be tractable. Standard assumptions on the cross-derivatives of utility are made, in order to ensure that the best responses are well defined and the equilibrium is unique.

Preferences. There is a continuum of agents indexed by $i \in [0,1]$. Each player chooses an action $k_i \in \mathbb{R}$. The average action in the population is $K := \int_0^1 k_i di$, the dispersion of individual actions in the population $\sigma_k := [\int_0^1 (k_i - K)^2 di]^{1/2}$. There is a payoff-relevant state $\theta \in \Theta \equiv \{0,1\}$, which is unknown with common prior belief μ_0. All players have the same payoff function

$$\begin{aligned} U : \mathbb{R} \times \mathbb{R} \times \mathbb{R}_+ \times \Theta \to \mathbb{R} \\ (k_i, K, \sigma_k, \theta) \mapsto u_i \equiv U(k_i, K, \sigma_k, \theta) \end{aligned}$$

Dispersion does not have first-order effects on utility, but only second-order nonstrategic effects.[2] This assumption, together with quadratic utility, implies that preferences can be written in the following form

$$U(k_i, K, \sigma_k, \theta) = (k_i, K, \theta)' \mathbf{M} (k_i, K, \theta) + U_{\sigma\sigma}\sigma^2/2$$

where $\mathbf{M}$ is a 3×3 matrix. Utility is concave in the individual action $U_{kk} < 0$ and has bounded cross derivative $\alpha := -U_{kK}/U_{kk} \in (-1,1)$.[3] Concavity ensures that the best response function is well defined, the bounds on the cross derivative ensure uniqueness of the decentralized equilibrium. Similar technical assumptions are imposed on utility so that the efficient best response is well defined and the efficient allocation is unique.[4] Other than these requirements, the setting accommodates many possible models, including models of strategic complements ($U_{kK} > 0$) and strategic substitutes ($U_{kK} < 0$).[5] Moreover, the assumption that utility is quadratic can be interpreted as a second-order approximation of more general utilities.

[2] That is $U_\sigma(k, K, 0, \theta) = 0$ and $U_{k\sigma} = U_{K\sigma} = U_{\theta\sigma} = 0$ for all (k, K, θ).

[3] The restriction that $-U_{kK}/U_{kk} > -1$ is not common in the literature, but is actually sufficient for uniqueness of a linear solution along the lines of the proof in Morris and Shin (2002) when the action space is bounded. Without this assumption it is not obvious that higher-order beliefs converge to zero even for a bounded action space. The same assumption can also be found in Huo and Pedroni (2020), but not in Bergemann and Morris (2013).

[4] In terms of primitives, concavity of welfare reads $U_{kk} + 2U_{kK} + U_{KK} < 0$ and $U_{kk} + U_{\sigma\sigma} < 0$. As it will be clear later, this restriction implies that the "first-best" allocation is optimal from the perspective of the planner. Also, to ensure uniqueness of the centralized equilibrium, the cross derivative of welfare has to be bounded. In terms of primitives, this reads $\alpha^\star := 1 - \frac{U_{kk} + 2U_{kK} + U_{KK}}{U_{KK} + U_{\sigma\sigma}} \in (-1,1)$. This assumption implies that the efficient (or "second-best") allocation is unique.

[5] See for example Angeletos and Pavan (2007) for some applications of this environment.

Information. Information is incomplete as agents do not observe directly the state θ, but hold a common prior belief $\mu_0 \in \Delta(\{0,1\})$ about it. Before taking actions, agents observe two signals x_i and y that are informative about the state. The distributions that generated those signals are common knowledge. The first signal x_i is private information and treated as exogenous. Its distribution is a mapping from the state to the signal space $\pi^x : \Theta \to \Delta(\{0,1\})$.[6] Assume that the private signal is correct with probability $p \in (1/2, 1)$, that is $\pi^x(x_i = 1|\theta = 1) = \pi^x(x_i = 0|\theta = 0) = p \; \forall i$. The second signal y is public information, and its distribution π^y is chosen endogenously by the planner, as explained in the following sections. I will assume that also the planner is potentially subject to an information friction, which implies that the public signal is a mapping from information available to the planner to the signal space: $\pi^y : S \to \Delta(\{0,1\})$. Planner's information set is given by the realization of a private signal $s \in S \equiv \{0,1\}$, correct with probability $q \in (1/2, 1]$, that is $\pi^s(s = 1|\theta = 1) = \pi^s(s = 0|\theta = 0) = q$. This specification of the information set for the planner nests the the possibility that the planner observes the state ($q = 1$) and that the planner is equally informed as agents ($q = p$) as special cases.

Decentralized Equilibrium. After observing the signals, agents update their expectations using Bayes rule and choose actions k_i to maximize their utility, taking aggregates (K and σ_k) and information (x_i and y) as given. The concept of equilibrium used here is the standard symmetric Bayes-Nash equilibrium. The action that each agent takes is given by

$$k(x, y) = \mathbb{E}_{\theta\sim\mu_0}[(1 - \alpha)\kappa(\theta) + \alpha K(\theta, y)|x, y] \qquad \text{(Best response)}$$

where $\kappa(\theta) := \kappa_0 + \kappa_1\theta$ denotes the complete-information equilibrium action,[7] $K(\theta, y)$ is the incomplete-information average action, and $\alpha := U_{kK}/|U_{kk}|$ denotes the equilibrium degree of coordination. The characterization of the equilibrium allocation and comparative statics can be found in appendix A.

Communication Policy. The only tool available to the planner in this setting is a communication policy, which is the distribution π^y of the public signal y. The policy is chosen with commitment to maximize ex-ante welfare $\mathbb{E}\, u(\pi^y)$

$$\mathbb{E}\, u(\pi^y) := \mathbb{E}_{\theta\sim\mu_0}\, \mathbb{E}_{x\sim\pi^x(\theta)}\, \mathbb{E}_{s\sim\pi^s(\theta)}\, \mathbb{E}_{y\sim\pi^y(s)}\, U\big(k(x, y), K(\theta, y), \sigma_k(\theta, y), \theta\big)$$

where $k(x, y)$ is the equilibrium allocation as a function of agents' information set, $K(\theta, y) \equiv \sum_x k(x, y)\pi^x(x|\theta)$ is the incomplete-information average action, and $\sigma_k(\theta, y) \equiv \big(\sum_x (k(x, y) - K(\theta, y))^2 \pi^x(x|\theta)\big)^{1/2}$ is the incomplete-information cross-sectional dispersion for all (θ, y).

[6]Since the state is binary, it is without loss of generality to work with a binary signal space, that is realizations can take a high or low value $x_i \in X \equiv \{0,1\}$.

[7]Where the scalars κ_0 and κ_1 are given by $\kappa_0 := -U_k(0,0,0,0)/(U_{kk} + U_{kK})$ and $\kappa_1 := -U_{k\theta}/(U_{kk} + U_{kK})$. Normalizing $\kappa_0 = 0$ is without loss, as the policy does not depend on it.

The optimal communication policy π^y is therefore obtained as a solution of the following problem

$$\max_{\pi^y} \mathbb{E}\, u(\pi^y) \qquad \text{(Planner's problem)}$$

The timing of the choice of the policy is as follows:

1. The planner chooses the communication policy $\pi^y : S \to \Delta(\{0,1\})$.
2. Agents observe which policy was chosen.
3. Nature draws randomly θ according to the common prior belief μ_0.
4. Nature draws an identically and independently distributed realization of the private signal x_i according to $\pi^x(\theta)$ for each $i \in [0,1]$.
5. Nature draws a realization of the private signal s according to $\pi^s(\theta)$.
6. Nature draws a realization of the public signal y according to $\pi^y(s)$.
7. Agents observe (x_i, y) and update their beliefs using Bayes rule.
8. Agents take actions k_i for all $i \in [0,1]$ and payoffs realize.

Externalities. In the absence of externalities, the optimal policy prescribes a full revelation of all the available information by the planner. In fact, an increase in the information provided to agents can only increase welfare, by attenuating the negative effects of the information friction. As common in the literature, I will introduce two stylized sources of externalities, which are responsible for possible wedges between the equilibrium and efficient allocations. These externalities make the choice of the optimal communication policy non-trivial, and moreover they encompass a large number of applications as special cases. From Angeletos and Pavan (2007), we know that the efficient (second-best) action is the solution of the following equation

$$k^\star(x,y) = \mathbb{E}_{\theta\sim\mu_0}[(1-\alpha^\star)\kappa^\star(\theta) + \alpha^\star K(\theta,y)|x,y] \qquad \text{(Efficient response)}$$

where $\kappa^\star(\theta) := \kappa_0^\star + \kappa_1^\star\theta$ denotes the first-best allocation,[8] and $\alpha^\star := [U_{\sigma\sigma} - 2U_{kK} - U_{KK}]/[U_{KK} + U_{\sigma\sigma}]$ denotes the efficient degree of coordination. The latter is the degree of coordination that the planner would like agents to perceive for their equilibrium actions to maximize ex-ante welfare. Details and derivations are in appendix A. Comparing equation (Best response) to equation (Efficient response) shows that the equilibrium action equals the efficient action if and only if both $\kappa = \kappa^\star$ and $\alpha = \alpha^\star$. I will refer to the economy being efficient when $\kappa = \kappa^\star$ (and inefficient otherwise), and to the economy displaying a coordination externality when $\alpha \neq \alpha^\star$. An example of an efficient economy with a coordination externality is the setting from Morris and Shin (2002), whereas an example without coordination externality in an inefficient economy can be found in Angeletos et al. (2016). An example with both is the application in section 3.

[8]With $\kappa_0^\star \equiv -\frac{U_k+U_K}{U_{kk}+2U_{kK}+U_{KK}}$ evaluated at $(0,0,0,0)$, and $\kappa_1^\star \equiv -\frac{U_{k\theta}+U_{K\theta}}{U_{kk}+2U_{kK}+U_{KK}}$.

2.1 Optimal Use of Public Information

Before discussing the solution of planner's problem, let us introduce some terminology. Given the assumption of binary state, the communication policy can only fall under one of the following three cases: a complete disclosure of the private signal s, a partial disclosure, or no disclosure.

Definition 1 (Communication Policies)**.** *A communication policy* π^y *is*

- *Fully revealing if* $\pi^y(y=1|s=1) = \pi^y(y=0|s=0) = 1$.
- *Partially revealing if* $\frac{1}{2} < \pi^y(y=1|s=1), \pi^y(y=0|s=0) < 1$.
- *Non-disclosing if* $\pi^y(y=1|s=1) = \pi^y(y=0|s=0) = \frac{1}{2}$.

In terms of interpretation, these three types of policies are very different. Whereas full revelation and not disclosure imply a degenerate distribution of posterior beliefs–conditional on the realization of the planner's private signal,– partial revelation leads to a non-degenerate distribution of posterior beliefs, which implies sunspot equilibria generated by the policy. One might expect welfare not to benefit from such non-fundamental fluctuations, hence the optimal policy to be only fully revealing or non-disclosing. We will see that this intuition is not correct, and explore implications in section 3.

Let us now move to the characterization of the optimal public communication policy by solving problem (Planner's problem) in steps. Using a key insight from the information design literature,[9] the problem can be recast as a maximization over the distribution τ of posterior beliefs μ_y that are induced by the policy. This approach allows to overcome a major difficulty in the determination of the optimal public signal, as in the original problem the maximization is over the function, whereas in the simplified problem it is over scalars. With a slight abuse of notation,[10] denote with $\mathbb{E}\, u(\mu_y) := \mathbb{E}_{\theta\sim\mu_y}\, \mathbb{E}_{x\sim\pi^x(\theta)}\, U\big(k(x,\mu_y), K(\theta,\mu_y), \sigma_k(\theta,\mu_y), \theta\big)$ the ex-ante welfare as a function of the posterior belief induced by the policy.

Lemma 1 (Planner's problem)**.** *The planner's problem in (Planner's problem) is equivalent to:*

$$\max_{\tau} \mathbb{E}_{\mu_y\sim\tau}\, \mathbb{E}\, u(\mu_y)$$

subject to

$$\mathbb{E}_{\mu_y\sim\tau}\, \mu_y = \mu_0 \qquad \text{(Plausibility)}$$

$$\mu_y \in [\mu_y^-, \mu_y^+] \qquad \text{(Information aggregation)}$$

where

$$\mu_y^- := \frac{(1-q)\mu_0}{(1-q)\mu_0 + q(1-\mu_0)}; \quad \mu_y^+ := \frac{q\mu_0}{q\mu_0 + (1-q)(1-\mu_0)}$$

[9]One can think about the reformulation of the problem as an application of the revelation principle. See Kamenica (2019) for a survey.

[10]In the appendix, welfare as a function of π^y is denote with a hat. Also k, K, σ_k as functions of y are denoted with a hat.

Interpretation of the Constraints. The plausibility constraint is standard in the literature, and imposes that the distribution of posterior is "Bayes plausible," which means that posteriors are computed using Bayes rule and the public signal has a proper distribution that integrates to one. A formal definition can be found in the proof of the lemma in the appendix. This can be thought as consistency requirement for the simplified problem.

The novelty of the problem is the information aggregation constraint, which shows up in the problem because the planner observes only a–possibly imperfect–signal about the state. Since this imperfect knowledge is understood by agents, they internalize the additional noise in the public signal that comes from the randomness of the private signal received by the planner. Hence, the same public signal induces different posteriors depending on q, and in particular the smaller q, the closer the posterior is to the prior. In other words, the length of the interval (μ_y^-, μ_y^+) is increasing in the precision q of the information available to the planner. For $q = 1$, the problem reduces to a standard information design problem, whereas in the limiting case with $q \to \frac{1}{2}$, the interval collapses to the prior belief μ_0.

In terms of interpretation, the information aggregation constraint captures the imperfect ability of the planner to collect disaggregated information. If the planner was able to directly communicate[11] with agents and collect their private signals, it would be as if the planner was able to observe the state, that is $q = 1$. However, in this setting, information frictions prevent the planner from being able to access such a knowledge, and thus impose a limit on the effectiveness of public communication.

Several rationalizations for this are possible. First, one can think about the information aggregation constraint as a technological restriction on the ability of the planner to forecast fundamentals; this is a natural interpretation in the case of a central bank trying to forecast macroeconomic fundamentals. Second, such a constraint would arise from an information acquisition problem in which the planner has to pay a cost to access more precise knowledge, which could be modelled along the lines of Colombo et al. (2014). In the static setting studied here, the constraint can be kept exogenous, but in dynamic settings q may depend on θ as in Mäkinen and Ohl (2015), Benhabib et al. (2016a), and Flynn and Sastry (2020). Finally, one can also think about the constraint in terms of credibility of the planner. The idea is that knowledge of fundamentals is relevant only because it adds credibility to the public signal, and this moves beliefs away from the prior. Nevertheless, even absent such a knowledge, as long as posteriors are different from the prior, communication is still effective. This intuition is related to the work on credibility and persuasion by Mathevet et al. (2019).

[11] In this case, a natural question would be whether the planner can improve upon the public signal using a direct communication with agents. Though interesting, answering this question is particularly challenging and would require a separate treatment.

Understanding the Tradeoffs We will now see that the optimal policy can be fully revealing, partially revealing, or non-disclosing depending on the types of economies analyzed and the quality of the private and public information available. For clarity of exposition, economies will now be classified along two dimension. The first dimension is whether the economy is efficient ($\kappa = \kappa^\star$), or inefficient ($\kappa \neq \kappa^\star$). The second dimension is the precision q of the private information available to the planner. In order to understand separately the tradeoffs, I will study how the policy changes for different values of q in efficient economies, whereas efficient and inefficient economies will be compared in the extreme case of $q = 1$ (no information aggregation constraint). Since efficient economies are a special case of inefficient ones, all the considerations under $q < 1$ apply also to inefficient economies.

2.1.1 Optimal Communication in Efficient Economies ($\kappa = \kappa^\star$).

Let us now move to the characterization of the optimal policy in efficient economies.

Theorem 1 (Efficient Economies with large q). *In economies with $\kappa = \kappa^\star$, there exists a $\bar{q} < 1$ such that for all $q \geq \bar{q}$, the optimal communication policy is fully revealing.*

The theorem shows that, in efficient economies, an optimal communication policy always reveals precise information when available. The limit to the quality of information that can be delivered by a communication policy is captured by the information aggregation constraint, which for q large enough always binds. This also means that precise information always increases welfare, even in those cases in which imprecise information may lower it. When the planner is able to collect sufficiently precise information about the state of the economy, in efficient economies there is no reason not to reveal that knowledge.

The proof is intuitive and general. Since $\kappa^\star$ is the first best, and $\kappa = \kappa^\star$, when $q = 1$ the planner can implement the first best by revealing the state, and thus fully resolving uncertainty.[12] Moreover, there is no discontinuity at $q = 1$, and in fact the same logic applies as long as the reduction in uncertainty is sufficiently large.

The threshold $\bar{q}$ is potentially a function of all the deep parameters of the model. In particular, it depends on the prior belief μ_0, the coordination wedge $\alpha^\star - \alpha$, and the precision of agents' private signal p. Nevertheless, turns out that there are very few cases in which q needs to be close to one for some form of revelation–full or partial–to be optimal. Moreover, for most calibrations, full revelation is optimal even when q is close to a half.[13] Intuitively, one such case is when $\alpha^\star \approx \alpha$, that is when the coordination externality is small. The

[12] This argument applies also when the information structure is restricted to be gaussian, as shown in Bergemann and Morris (2013) in the context of a Cournot game.

[13] This is related to the point raised by Svensson (2006) in the setting with information restricted to be gaussian.

next theorem explores in detail those cases and how the optimal communication changes when the planner has access to less precise information.

Theorem 2 (Efficient Economies with smaller q)**.** *In economies with $\kappa = \kappa^\star$, there exists a $\tilde{\mu}_0 \le 1/2$ such that:*

1. *For $\mu_0 \in [0, \tilde{\mu}_0] \cup [1-\tilde{\mu}_0, 1]$, there exists a $\bar{q} > \frac{1}{2}$ such that the optimal communication policy is non-disclosing for all $q \le \bar{q}$ if and only if*

$$\frac{\alpha^\star - \alpha}{1-\alpha^\star} > \frac{(2p-1)^2(1-\alpha) + p(1-p)}{(2p-1)^2(1-\alpha)} \ge 0 \qquad \text{(Concavity at } \{0,1\}\text{)}$$

2. *For $\mu_0 \in [\tilde{\mu}_0, 1-\tilde{\mu}_0]$, there exists a $\bar{q} > \frac{1}{2}$ such that the optimal communication policy is non-disclosing for all $q \le \bar{q}$ if and only if*

$$\frac{\alpha - \alpha^\star}{1-\alpha^\star} > \frac{(1-\alpha) + 4\alpha p(1-p)}{2(2p-1)^2(1-\alpha)} \ge 0 \qquad \text{(Concavity at } 1/2\text{)}$$

3. *For $\mu_0 \in [\tilde{\mu}_0, 1-\tilde{\mu}_0]$ there exist $\underline{q} > \frac{1}{2}$ and $\bar{q} < 1$ such that the optimal communication policy is partially revealing for all $q \in (\underline{q}, \bar{q})$ and fully revealing for all $q \notin (\underline{q}, \bar{q})$ only if condition (Concavity at $\{0,1\}$) holds.*

4. *For $\mu_0 \in [0, \tilde{\mu}_0] \cup [1-\tilde{\mu}_0, 1]$ there exist $\underline{q} > \frac{1}{2}$ and $\bar{q} < 1$ such that the optimal communication policy is partially revealing for all $q \in (\underline{q}, \bar{q})$ and fully revealing for all $q \notin (\underline{q}, \bar{q})$ only if condition (Concavity at $1/2$) holds.*

The theorem shows that, when private information available to the planner is imprecise, no disclosure or partial disclosure can be optimal. In particular, no disclosure is optimal if and only if either coordination is inefficiently low (i.e. $\alpha - \alpha^\star$ is negative and small) and prior uncertainty is sufficiently low (i.e. prior belief μ_0 is close to either zero or one), or coordination is inefficiently high (i.e. $\alpha - \alpha^\star$ is positive and large) and the prior uncertainty is high (i.e. prior belief is close to a half).[14]

These two cases (1 and 2) cannot happen at the same time, as conditions (Concavity at $\{0,1\}$) and (Concavity at $1/2$) are mutually exclusive. This implies that when either of those conditions holds and q is small, there are some values of the prior belief μ_0 for which the optimal policy is revealing, and others for which it is non-disclosing. Moreover, for q small, the optimal policy can only be either non-disclosing or fully revealing. When fully revealing at a small q, something interesting happens: as q increases, the policy can become partially revealing. In particular, while for both $q \le \underline{q}$ and $q \ge \bar{q}$ the optimal policy would actually be fully revealing, it is partially revealing at $q \in (\underline{q}, \bar{q})$. This highlights how, in a sense, the optimal policy needs not be "monotonic in q."

[14] As discussed in appendix A, a positive coordination wedge $\alpha^\star - \alpha$ implies that the cross-sectional dispersion of actions σ_k is excessively large (from planner's perspective), whereas a negative wedge implies that volatility of the wedge $K - \kappa$ is excessively large.

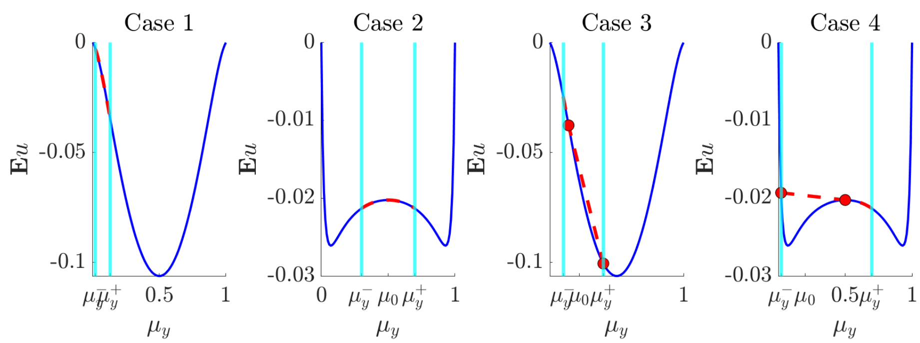

Figure 1: Illustration of the construction of the optimal communication policy for theorem (2). Welfare as a function of the posterior belief μ_y is in blue. Calibration for case 1 and 3 is $p = .7$, $\kappa_0 = 0$, $\alpha = -.9$, $\kappa_1 = 1$, $\alpha^\star = .9$, and $\kappa_1^\star = 1$. Calibration for case 2 and 4 is $p = .99$, $\kappa_0 = 0$, $\alpha = .8$, $\kappa_1 = 1$, $\alpha^\star = -.8$, and $\kappa_1^\star = 1$. Optimal policy is the red dashed line, bounds μ_y^- and μ_y^+ are in cyan.

The intuition for this result is that, in the presence of a coordination externality, uncertainty and information interact in a non-trivial way. Information may increase welfare when uncertainty is high but not when it is low, or viceversa, depending on the sign of the externality. For q small, the optimal policy must be of a bang-bang form, because the effect of a release of information on uncertainty is marginal. For q large, the policy is fully revealing, because a lot of information can resolve all the uncertainty, which is optimal in efficient economies. For q intermediate, revelation of information can lower uncertainty enough to make further disclosures harmful, but not enough to fully resolve uncertainty, and thus only a partial revelation is optimal. Therefore, in the presence of a coordination externality sufficiently large, the optimal policy may not be fully revealing. This is in line with the main insight from Morris and Shin (2002) that public information can lower welfare in the presence of this type of distortion.[15] In addition, the above theorems clarify how information can be used strategically and how its optimal use depends on the quality of information available to the planner, or equivalently credibility.

The proof of theorem (2) can be understood by looking at figure (1), which plots welfare as a function of the posterior belief induced by the public signal. From the plot, one can find the posterior beliefs that maximize welfare, and then obtain the signal that induced those posterior as a last step. The optimal communication is the red dashed line. A revealing policy induces two posterior beliefs $\mu_y,\ y \in \{0, 1\}$, both of them with strictly positive probabilities (the red dots in the figure). A non-disclosing policy sends an uninformative signal that

[15]For $q \to 1/2$, the policy is determined only by the curvature of welfare in the μ_y-space. The curvature captures the "local" value of information, which is in spirit a similar exercise to the comparative statics one from Morris and Shin (2002).

makes posterior beliefs μ_y equal to the prior belief μ_0 with probability one. In expectations across the realizations of posterior beliefs, a revealing policy achieves possibly higher welfare than a non-disclosing policy, e.g. when welfare is convex. The value of welfare that is obtained at the optimal policy can be read off the graph as the point on the red line corresponding to the prior belief μ_0. When $q = 1$, a fully revealing communication induces posteriors $\mu_0 = \mu_y^- = 0$ and $\mu_1 = \mu_y^+ = 1$, thus the optimal policy is a constant line at zero, which means that welfare is equal to zero for all prior beliefs. When the policy is further constrained by $q < 1$, a communication strategy can at most induce posteriors in $[\mu_y^-, \mu_y^+] \subset [0, 1]$. A convex welfare in the μ_y-space means that information is locally beneficial, or in other words that the optimal policy is revealing for $q \to 1/2$, whereas concavity implies a non-disclosing policy for $q \to 1/2$.[16] In the presence of a coordination externality, welfare can be everywhere convex (trivially fully revealing), concave-convex-concave (cases 1 and 3), or convex-concave-convex (cases 2 and 4). The main part of the proof is to show that welfare can only be one of these three cases. In particular, conditions (Concavity at $\{0, 1\}$) and (Concavity at 1/2) characterize the curvature of welfare around $\mu_y \in \{0, 1\}$ and $\mu_y = 1/2$.[17] The rest of the proof builds on this geometric intuition to construct the optimal policy as in the figure. When q is sufficiently small, the local value of information fully determines the optimal policy, which is non-disclosing if and only if welfare is concave (cases 1 and 2). For a larger q, if the prior belief is in the convex region, some revelation of information must be beneficial, but partial revelation may be preferred to full revelation in the presence of the coordination externality. Cases 3 and 4 of the theorem clarify that conditions (Concavity at $\{0, 1\}$) and (Concavity at 1/2) are necessary for partial revelation to be optimal, but they are not sufficient. In fact, one can construct examples in which the optimal policy is fully revealing and conditions are satisfied. For instance, under condition (Concavity at $\{0, 1\}$), the optimal policy is fully revealing for any possible value of q when the prior belief is close to a half. This is an example in which the threshold $\bar{q}$ from theorem (1) is actually equal to 1/2 independently of the coordination wedge.

Summary for Efficient Economies. Three principles stand out from the analysis of efficient economies. First, precise public information is always welfare improving. When the planner has sufficient knowledge to be precise and therefore communication is credible, there is no reason to strategically withhold information. Second, withholding information may be beneficial when the planner has only access to imprecise information and therefore communication is less powerful. In that case, the optimal communication is of the bang-bang type, either fully revealing or not disclosing depending on prior uncertainty and the direction of the coordination externality. Third, the optimal policy is not necessarily "monotonic in q," meaning that it can be fully revealing when both $q \to 1/2$ and $q \to 1$, but partially revealing at some intermediate values.

[16] See Kamenica and Gentzkow (2011) for the geometric intuition.
[17] The general proof is done for theorem (3), and theorems (1)-(2) are a special case.

2.1.2 Optimal Communication in Inefficient Economies ($\kappa \neq \kappa^\star$)

In inefficient economies, an additional tradeoff is present. The planner takes into account that public information reduces the gap between the average action and the action under complete information ($K - \kappa$).[18] When the action under complete information is inefficient ($\kappa \neq \kappa^\star$), reducing the gap may not be optimal. As shown in appendix A, reducing the gap lowers welfare through an increase in the inefficiency whenever the covariance between $K - \kappa$ and the inefficiency wedge $\kappa^\star - \kappa$ is positive.[19] In this case, less information generates an efficiency gain. On the other hand, a volatile gap also contributes negatively to welfare, and more information reduces its volatility. These two opposite forces imply that the optimal policy can be non-disclosing even when $q = 1$, that is even when public communication is mostly powerful. Hence, differently from efficient economies, in inefficient economies precise information need not be always welfare improving.

Theorem 3 (Inefficient Economies)**.** *Let $q = 1$. There exists a $\tilde{\mu}_0 \leq 1/2$ such that:*

- *For $\mu_0 \in [0, \tilde{\mu}_0] \cup [1 - \tilde{\mu}_0, 1]$, the optimal communication policy is revealing if*

$$\frac{2\kappa_1^\star - \kappa_1}{\kappa_1} \geq \left(\frac{\alpha^\star - \alpha}{1 - \alpha^\star}\right)\left(\frac{(2p-1)^2(1-\alpha)}{(2p-1)^2(1-\alpha) + p(1-p)}\right) \tag{Convexity at $\{0,1\}$}$$

  Moreover, it is revealing only if condition (Convexity at $\{0, 1\}$) holds or

$$\kappa_1^\star \geq \frac{1}{2}\kappa_1 \tag{Decreasing at 0}$$

- *For $\mu_0 \in [\tilde{\mu}_0, 1 - \tilde{\mu}_0]$, the optimal communication policy is revealing if*

$$\frac{2\kappa_1^\star - \kappa_1}{\kappa_1} \geq -\left(\frac{\alpha^\star - \alpha}{1 - \alpha^\star}\right)\left(\frac{2(2p-1)^2(1-\alpha)}{(1-\alpha) + 4\alpha p(1-p)}\right) \tag{Convexity at 1/2}$$

  Moreover, it is revealing only if condition (Convexity at 1/2) holds or

$$\frac{2\kappa_1^\star - \kappa_1}{\kappa_1} \geq \frac{(2p-1)^2}{1-\alpha^\star}\left[2\frac{\kappa_1^\star}{\kappa_1}(\alpha - \alpha\alpha^\star) - \alpha^\star - \alpha^2 + 2\alpha\alpha^\star\right] \tag{Negative at 1/2}$$

The theorem provides a full characterization of the optimal policy for the case $q = 1$. Conditions (Convexity at $\{0, 1\}$) and (Convexity at 1/2) characterize the curvature of welfare, respectively, close to $\mu_y \in \{0, 1\}$ and at $\mu_y = 1/2$.

[18] In applications this wedge is often referred to as the output gap. The reason for this is that κ corresponds to the symmetric action in the frictionless version of the model, whereas K is the average action in the model with frictions. If the information friction is the source of a nominal rigidity in price setting, then this gap is the traditional output gap in models with pricing rigidities. See the application in section 3 for an example.

[19] This is the case whenever $\kappa_1 < \kappa_1^\star$.

Condition (Decreasing at 0) implies that welfare is below zero close to the extrema of the domain, and condition (Negative at 1/2) implies that welfare is below zero in the center. The optimal communication is fully revealing if revealing for $\mu_0 \in [0,1]$, not disclosing if not disclosing for $\mu_0 \in [0,1]$, and partially revealing or not disclosing in the remaining cases, depending on whether the prior belief is close to a half or close to the extrema of its domain. It is easy to check that conditions (Convexity at $\{0,1\}$) and (Convexity at 1/2) can lead to either of the above configurations.

Corollaries (1)-(5) in appendix A break down the theorem in special cases and provide details on implementation of the optimal policy and comparative statics on the thresholds for the wedges $\kappa - \kappa^\star$ and $\alpha - \alpha^\star$. The main intuitions are summarized as follows. First, the theorem nests efficient economies as a special case. As shown before, the optimal communication is fully revealing. This follows because both condition (Decreasing at 0) and (Negative at 1/2) hold at $\kappa = \kappa^\star$. This provides an additional geometric insight that was not used in the proof of theorem (1). Second, when the inefficiency is sufficiently small ($\kappa_1^\star \approx \kappa_1$), the optimal policy is the same as in efficient economies, that is fully revealing. This is also the optimal policy when both condition (Convexity at $\{0,1\}$) and (Convexity at 1/2) are satisfied. On the other hand, when the inefficiency wedge is large and less information generates efficiency gains, the optimal communication is non-disclosing. This happens when neither condition (Convexity at $\{0,1\}$) nor (Convexity at 1/2) is satisfied. Third, for intermediate levels of the inefficiency, a partial revelation of information can be optimal. In particular, it is optimal either when coordination is inefficiently high (i.e. $\alpha - \alpha^\star$ is positive and large) and prior uncertainty is high (i.e. prior belief μ_0 close to zero or one), or when coordination is inefficiently low (i.e. $\alpha - \alpha^\star$ is negative and small) and prior uncertainty is low. The former case is when condition (Convexity at $\{0,1\}$) applies but not (Convexity at 1/2) or (Negative at 1/2), the latter is when condition (Decreasing at 0) applies but not (Convexity at $\{0,1\}$) or (Decreasing at 0). In all the other cases, not disclosing is optimal.

The intuition for the result is that, as the planner faces an additional tradeoff between efficiency gains and tackling the coordination externality, a full revelation of all the available information may not be optimal whenever this leads to large losses in terms of efficiency. This intuition is reminiscent of results from Angeletos and Pavan (2007), that showed that, in the presence of inefficiency, increasing the precision of a gaussian public signal may lower welfare. The most interesting economies are those in which the two forces balance each other, so that an interior solution with partial revelation is optimal.

The proof follows the same geometric logic as that of theorem (2). The main part is the characterization of the curvature of welfare in the μ_y-space. This is done in steps: first welfare is showed to be twice continuously differentiable, symmetric around 1/2, and with at most two zeros in the second derivative. Then welfare and its derivatives are evaluated at the extrema $\mu_y \in \{0,1\}$ and at $\mu_y = 1/2$. This gives the conditions for the theorem. Finally, the construction of the optimal policy follows from the curvature of welfare and can be visualized in figure (2). There are four cases, depending on which combination of the two

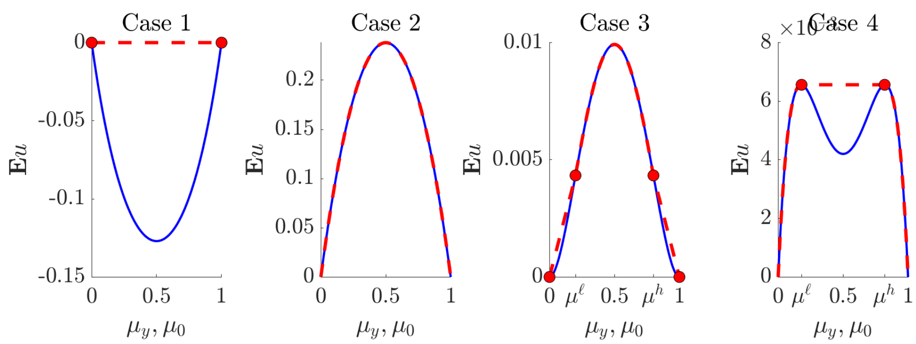

Figure 2: Illustration of the construction of the optimal communication policy for theorem (3). Welfare as a function of the poster belief μ_y is in blue. Calibration is $p = .7$, $\kappa_0 = 0$, $\kappa_1 = 1$ for all four cases. For the remaining parameters: $\alpha =$ (i) .5, (ii) .5, (iii) .5, (iv) 0; $\kappa_1^\star =$ (i) .8, (ii) 0, (iii) .5, (iv) .5; $\alpha^\star =$ (i) 0, (ii) 0, (iii) 0, (iv) .4. Optimal policy is the red dashed line.

sufficient conditions for the theorem are satisfied. Since $q = 1$, the information aggregation constraint is never binding and $\mu_y^- = 0$, $\mu_y^+ = 1$. The local value of information can be understood as concavity/convexity patterns of welfare as a function of the posterior belief μ_y. When welfare is convex, a revealing communication is generally optimal. When welfare is convex over the whole domain, the optimal policy is fully revealing and achieves constant welfare of zero regardless of the prior belief μ_0. In this case, the value of information is always positive. On the other hand, when welfare is concave, the value of information is negative and the optimal policy prescribes no disclosure. The intermediate cases described above can be visualized in cases 3 and 4. Welfare is convex-concave-convex and concave-convex-concave respectively, which implies that for some values of the prior belief μ_0 the optimal policy is partially revealing, whereas for the other values of μ_0 the optimal communication is non-disclosing.

Summary for Inefficient Economies. When less information generates efficiency gains, there is a new tradeoff to be considered when designing the optimal communication policy. This new force implies that the optimal communication need not be fully revealing even in the absence of the information aggregation constraint. For small gains, the optimal policy is the same as in efficient economies, and previous considerations apply. On the other hand, for a large inefficiency, non-disclosing information is optimal. The tradeoff is mostly interesting in the intermediate cases, when efficiency gains are of the same order as losses from the coordination externality. In this case, a partial revelation of information can be optimal, depending on the direction of the externality and the prior level of uncertainty. In particular, the communication is partially revealing if and only if coordination is inefficiently high and prior uncertainty is large, or coordination is inefficiently low and prior uncertainty is small.

3 Illustration: Central Bank Communication

As a main illustration, this section derives the optimal communication policy in a fully microfounded business-cycle model with nominal rigidities arising from incomplete information and pricing complementarities, as in previous works by Lucas (1972), Hellwig (2005), Adam (2007), Roca (2010), and Lorenzoni (2010). The model abstracts from optimal interest rate policy for clarity of exposition, but one could extend the baseline analysis to include traditional monetary policy, e.g. along the lines of Angeletos et al. (2016), Angeletos and La'O (2020), and Kohlhas (2020).

Motivation. To see why this connection is a timely point, consider the literature on "Delphic" central bank communication. A number of empirical papers, including Campbell et al. (2012), Nakamura and Steinsson (2018), Jarociński and Karadi (2020), Andrade and Ferroni (2021), and Leombroni et al. (2021), documented that there are in fact substantial real effects of information releases by the monetary policy authority. This can be a particularly important tool in the presence of a zero-lower bound, as emphasized in Coenen et al. (2017). It is not obvious empirically whether this effect can be interpreted as being caused by a superior knowledge of economic fundamentals that the central bank has, thus a direct update of expectations of the private sector. However, in practice, it is hard to believe that financial investors have in fact so much worse information than the central bank to rationalize the large impacts observed on both financial markets and real economy. Coordination under incomplete information could provide a realistic setting to think about these issues. In turn, the power of central bank communication would not be interpreted as coming solely from a direct update of expectations, but also from the fact that it is public information that the private sector can use to coordinate upon. This is the sense in which public information differs from private information.

3.1 A Stylized Model of the Business Cycle

The economy is populated by a representative household, a continuum of monopolistically competitive firms, a final good producer, and a monetary authority–the central bank. The household is subject to a cash-in-hand constraint. Firms take pricing decisions subject to an information friction, whereas labor adjusts flexibly to clear the market.

Household. The representative household enjoys utility from consumption and disutility from working. Utility of the representative household is given by

$$\mathcal{U}_t \equiv \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} (\log C_{t+\tau} - N_{t+\tau})$$

where C_t is the aggregate consumption bundle at time t and N_t is the aggregate labor supply. The aggregate consumption bundle is obtained by aggregating intermediate goods c_{it} with the usual Dixit-Stiglitz aggregator with elasticity of substitution $\eta > 1$[20]

$$C_t = \left[\int_0^1 c_{it}^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}$$

Utility is maximized subject to the budget constraint

$$P_t C_t = (1-\tau) W_t N_t + \Pi_t + T_t$$

where P_t is the aggregate price index, W_t is the nominal labor wage, τ is a non-contingent labor tax, Π_t are total profits of the firms, and $T_t = \tau W_t N_t$ is a lump-sum transfer. In addition, the household is subject to a cash-in-hand constraint

$$P_t C_t = M$$

where M is the aggregate money supply. The money supply is assumed to be fixed over time as in Hellwig (2005) and Colombo et al. (2014).[21]

Final Good Producer. The final good producer produces the aggregate bundle with a linear production technology. Inputs are purchased from firms, that produce the differentiated intermediate goods. Profit maximization leads to the usual downward-sloping demand curve for variety i

$$c_{it} = \left(\frac{p_{it}}{P_t} \right)^{-\eta} C_t$$

where p_{it} is the price of variety i and the aggregate price index is

$$P_t = \left[\int_0^1 p_{it}^{1-\eta} di \right]^{\frac{1}{1-\eta}}$$

Firms. Firm i produces the differentiated variety with technology $c_{it} = A_t n_{it}^{\frac{1}{\omega}}$, where A_t is aggregate productivity, n_{it} is labor demand of firm i, and $\omega > \frac{2\eta-2}{2\eta-1}$ can accommodate constant returns to scale, decreasing returns when larger than one, or increasing returns when smaller.[22] Firms maximize profits subject to an information friction, which shows up in the expectation operator that is indexed by i. Prices are set on the basis of imperfect knowledge (nominal rigidities), whereas labor is free to adjust after uncertainty resolves and clear the market.

[20] The model allows for markup shocks arising from time-varying elasticity of substitution $\eta_t > 1 \ \forall t$. A microfoundation is provided in appendix B. In the baseline framework, the elasticity shows up in the coordination parameter α (independent of t), hence the more cumbersome model with islands in the microfoundation.

[21] Such rule seems plausible if the time horizon is sufficiently short, but need not be optimal. Any serious analysis of optimal monetary policy requires a richer setting, which is beyond the scope of this article.

[22] This restriction on ω implies $\alpha \in (-1, 1)$ and $\alpha^\star \in (-1, 1)$, but also $\alpha^\star > \alpha$. See appendix D for a different microfoundation that allows for $\alpha^\star \leq \alpha$.

The problem of the firm is

$$\max_{p_{it},c_{it}} \ \mathbb{E}_{it}\left\{p_{it}c_{it} - W_t n_{it}\right\}$$
$$\text{s.t. } c_{it} = \left(\frac{p_{it}}{P_t}\right)^{-\eta} C_t \qquad \text{(Firm's problem)}$$

Business Cycle. Two sources of fluctuations will be studied in this environment: productivity shocks and aggregate markup shocks. Both productivity and aggregate markup are log-linear in a binary fundamental $\theta_t \in \{0, 1\}$, which evolves as a Markov process. I will assume that the fundamental is unknown at the beginning of each period, and the prior belief μ_0 conditional on past realizations is common for all agents, that is $\theta_t|\theta_{t-1} \sim \mu_0$. Productivity A_t is a function of the state $\log A(\theta_t) = \kappa_0^A + \kappa_1^A\theta_t$, $\kappa_0^A, \kappa_1^A \geq 0$, the aggregate markup $\mathcal{M}_t$ is countercyclical and depends on the state $\log \mathcal{M}(\theta_t) = -(\kappa_0^{\mathcal{M}} + \log\omega + \kappa_1^{\mathcal{M}}\theta_t)$, $\kappa_0^{\mathcal{M}}, \kappa_1^{\mathcal{M}} \geq 0$. A microfoundation for the markup shock can be found in appendix B.

Information. Agents do not observe directly the fundamental θ_t, but receive signals that are informative about it. Private information available to firm i is given by the realization of a private signal x_{it}, correct with probability $p \in (1/2, 1)$. Also, all the agents observe the realization of an endogenous public signal $y_t \in Y$, and the history of aggregate price and quantities produced.[23] Therefore, the information set of firm i is given by $\mathcal{I}_{it} = \left\{x_{it}, y_t, \{C_\tau, P_\tau, y_\tau\}_{\tau=0}^{t-1}\right\}$ and the conditional expectation $\mathbb{E}_{it}$ denotes expectations conditional on $\mathcal{I}_{it}$.[24] Since prices p_{it} are subject to a nominal friction, they are measurable with respect to $\mathcal{I}_{it}$, whereas labor demand n_{it} is free to adjust and thus measurable with respect to θ_t. In addition, the central bank observes a private signal $s_t \in S$, correct with probability $q \in (1/2, 1]$.

Optimal Policy. The central bank chooses optimally a public communication policy $\pi^y : S \to \Delta(Y)$, which is a mapping between private information available to the monetary authority to a distribution over the public signal space. The choice of the communication policy is made under commitment to maximize ex-ante household's welfare $\mathbb{E}_{t-1}\mathcal{U}_t$, where $\mathbb{E}_{t-1}$ denotes the expectation conditional on the information set at the beginning of period t, before signals are received. The problem of the central bank at the beginning of time t is

$$\max_{\pi^y} \ \mathbb{E}_{t-1}\mathcal{U}_t(\pi^y) \qquad \text{(Central Bank's Problem)}$$

The timing of the choice is as follows: first, the central bank commits to a communication strategy π^y, that is commits to sending a public signal y_t to

[23] In this model, observing the history implies that at the beginning of period t, agents know θ_{t-1}.

[24] The representative household's information set is given by all the realizations of the private signals $\mathcal{I}_t = \cup_{i\in I}\mathcal{I}_{it}$. This implies that aggregates are measurable with respect to θ_t.

the other agents upon observing the realization s_t. In practice, this amounts to choosing two probabilities, of sending a high signal when private information indicates that the state is high $\pi^y(y_t = 1|s_t = 1)$, and of sending a low signal conditional on a bad realization of the private signal $\pi^y(y_t = 0|s_t = 0)$. Given the assumption of binary state, these two numbers pin down the full distribution of y_t. After this choice is made, the state θ_t realizes and signal $\{x_{it}\}_{i\in I}, s_t$ are sent. Then, the communication takes place and the public signal $y_t \sim \pi^y(s_t)$ is received by agents. Finally, firms choose prices p_{it} based on their information set $\mathcal{I}_{it}$, and labor supply n_{it} clears the market.

The non-contingent tax is chosen such that, absent fluctuations in the aggregate markup, the economy is efficient, that is $\log(1+\tau) = -\kappa_0^{\mathcal{M}}$.

3.2 Decentralized Equilibrium and Welfare

As shown in appendix B, the first-order condition for the problem (Firm's problem) is

$$p_{it}^{1+\eta\omega-\eta} = \frac{\mathbb{E}_{it}\left[\omega \mathcal{M}_t M^\omega P_t^{\omega(\eta-1)} A_t^{-\omega}\right]}{\mathbb{E}_{it}\left[P_t^{\eta-1}\right]} \tag{Firm's FOC}$$

where $\mathcal{M}_t$ is the markup net of labor taxes. Log-linearizing around the complete-information allocation evaluated at $\theta_t = 0$, it is easy to show that, up to the first-order approximation, the price satisfies

$$\log p_{it} = (1-\alpha)\,\mathbb{E}_{it} \underbrace{\left\{\frac{1}{\omega}\log(\omega\mathcal{M}_t) + \log(M/A_t)\right\}}_{\text{Reset Price}} + \alpha\,\mathbb{E}_{it}\log P_t$$

where $\alpha := \frac{(\omega-1)(\eta-1)}{1+\eta\omega-\eta} \in (-1,1)$ is the optimal level of coordination. The equation states that, absent the information friction, the price that firms choose is a markup over the real marginal cost M/A_t (as $W_t = W = M$); when firms do not directly observe fundamentals, and thus need to forecast $\mathcal{M}_t$ and A_t, the optimal price is a linear combination of the expectation of the "reset price" and the aggregate price in the economy. The latter enters with a positive weight provided that $\omega > 1$, that is the model displays strategic complementarities in pricing whenever there are decreasing returns to scale in production.

Let us denote with $\hat{x}_t := \log C_t - \log \tilde{C}_t$ is the output gap, that is the difference between the log aggregate consumption $\log C_t$ and the log aggregate consumption in the frictionless benchmark (the complete-information allocation) $\log \tilde{C}_t$. Also the cross-sectional dispersion in prices is given by $\sigma^2_{\log p_{it}} \equiv \int_0^1 (\log p_{it} - \log P_t)^2 di$. Under a standard "small distortion" assumption, expanding welfare $\mathbb{E}_{t-1}\mathcal{U}_t$ up to second order around the complete-information allocation and ignoring term that are independent of the communication policy leads to a familiar representation in terms of three components: covariance, volatility, and dispersion.

$$\mathbb{E}_{t-1}\,\mathcal{U}_t \approx \mathbb{E}_{t-1}\sum_{\tau=0}^{\infty}\beta^{\tau}\left\{\left(1-\frac{1}{\mathcal{M}_\tau}\right)\hat{x}_\tau - \frac{\omega}{2}\hat{x}_\tau^2 - (\eta-1-\eta\omega)\frac{\eta}{2}\sigma^2_{\log p_{i\tau}}\right\} \tag{Household's welfare}$$

The first term $\mathbb{E}_{t-1}\left\{\left(1-\frac{1}{\mathcal{M}_t}\right)\hat{x}_t\right\} \approx \mathrm{Cov}_{t-1}\Big(\log\mathcal{M}_t, \hat{x}_t\Big)$ is the covariance between the aggregate markup (in logs) and the output gap. When the covariance is positive (countercyclical markup), it means that deviations of production from the frictionless benchmark move the economy towards efficiency. The second term $\mathbb{E}_{t-1}\,\hat{x}_t^2$ is the volatility of the output gap; this term captures the extent to which aggregate production differs from the production level that firms would optimally choose if they were not subject to the information friction. The term is often referred to in the literature as "non-fundamental volatility," because differences between the aggregate production and the production under complete information are solely driven by the interaction between complementarities and imperfect information, hence they are not driven by fundamentals.[25] The last term $\mathbb{E}_{t-1}\,\sigma^2_{\log p_{it}}$ is the cross-sectional dispersion of prices; with nominal frictions à la Calvo, price dispersion would map directly into inflation,[26] but more generally it captures output losses generates by inefficient dispersion in prices.

Which term of the three is mostly relevant for welfare depends on the inefficiency and coordination externality. The importance of the inefficiency is determined by the covariance between the aggregate markup and the state $\mathrm{Cov}(\log\mathcal{M}_t, \theta_t) = -\kappa_1^{\mathcal{M}} \leq 0$. From planner's perspective, any covariance is harmful, that is the efficient level of covariance is given by $\mathrm{Cov}(\log\mathcal{M}_t^{\star}, \theta_t) = \kappa_1^{\star\mathcal{M}} = 0$.[27] The smallest the wedge $\kappa_1^{\mathcal{M}} - \kappa_1^{\star\mathcal{M}} \leq 0$, the largest the inefficiency, and thus the largest the welfare gains from the covariance term. On the other hand, the relative importance of volatility and dispersion depends on the coordination externality. The efficient level of coordination is given by $\alpha^\star := \frac{1+\eta\omega-\eta-\omega/\eta}{1+\eta\omega-\eta} \in (-1,1)$. The direction of the externality is therefore

$$\alpha^\star - \alpha = \frac{\omega(\eta-1)}{\eta(1+\omega\eta-\eta)} > 0$$

The term being positive means that firms would be better off if they were to perceive stronger pricing complementarities, or, in other words, dispersion is inefficiently high compared to volatility. In terms of policy, this means that the central bank is willing to let the output gap fluctuate–thus generate welfare losses for the household–in order to reduce the inefficiently high cross-sectional dispersion in prices (or inflation, in models with rigidities à la Calvo).

[25] See for example Angeletos and La'O (2013).

[26] I will loosely refer to dispersion as inflation, with the understanding that standard microfoundations provide the link between the two. See Woodford (2003), chapter 6.

[27] Notice that, because of the fiscal labor tax, $\kappa_0^{\mathcal{M}} + \log(1+\tau) = \kappa_0^{\star\mathcal{M}} = 0$. Relaxing this assumption would not change the optimal communication policy, because communication cannot affect the level of the economy, only fluctuations around it.

3.3 Tradeoffs in Communication

A communication strategy by the central bank is a policy of the following form: the central bank collects information about the state of economy, and then conditional on the outcome of that research has the option to disclose the knowledge acquired. The assumption of commitment, which is here a timing assumption, is crucial. The choice of whether to disclose the outcome of the research is made before actually collecting the information and observing the results, and then it is written in the central bank's mandate so that agents internalize it. Once this choice is made, then the research takes place, and the outcome is disclosed to the public or not according to the mandate.

Three forms of communication are possible: a non-disclosing policy does not reveal any information; a fully revealing policy simply publishes the outcome of the research; a partially revealing policy is a coin toss between disclosing and not disclosing, conditional on the outcome of the research. Therefore a partially revealing communication allows potentially for full revelation if the outcome of the research is favorable, and a coin toss if the outcome is unfavorable. It is clear that, for the policy to have any effect on firms' beliefs, it cannot be that a policy disregards completely the outcome of the research, and always tries to influence firms towards believing the favorable outcome. Since agents are aware of the mandate, such a communication would be ineffective. Therefore, even if the central bank wants to influence agents' beliefs in one direction, the probability of the coin toss has to be chosen such that there is sufficient information content in any announcements so that agents still want to listen to them.

A revealing policy always entails to some extent an increase in the information available to agents. Public information has two effects, through coordination and uncertainty. On the one hand, more information increases the degree of coordination among firms. In fact, absent the information friction, all firms would perfectly coordinate and choose the same price. Moreover, public information is particularly effective in fostering coordination as firms can use it to forecast what other firms are doing as well. In turn, this reduces the inefficient dispersion in their prices, increases output and, in a dynamic setting with nominal rigidities à la Calvo, reduces inflation. On the other hand, more information has an effect on the uncertainty that firms hold about the state of the economy, i.e. in this case the realization of shocks. A revealing policy does not necessarily reduce uncertainty; in fact, uncertainty is reduced when prior beliefs held by agents are confirmed, but possibly increased when the outcome of the research goes in the opposite direction as what agents thought the state of the economy to be. To understand this channel consider the following example. Suppose that, before any information about productivity is revealed through market interactions and public communication, firms believe that a recession (negative productivity shock) is very likely. Suppose also that, before conducting the research, the central bank committed to a fully revealing policy, which amounts to the publication of the research independently of the result. If the outcome of the research confirms the recession, uncertainty is further reduced; this reduces dispersion in prices and inflation. If the outcome of the research indicates an expansion, whether uncertainty goes up or down depends on the credibility of the research. For a sufficiently credible analysis, uncertainty will go down as well after its publication, because firms will believe it and coordinate their pricing decisions as in an expansion. However, when the analysis is not fully convincing, its publication will confuse firms and uncertainty will increase because the evidence is conflicting with prior beliefs. This in turn would increase dispersion in prices and inflation.

The optimal communication policy balances optimally the effects on coordination and uncertainty. It follows that it must depend on the prior level of uncertainty of agents of the economy, as well as the precision of the information that the central bank is able to collect. In the model, the former is described by the prior belief μ_0 and the latter by the information aggregation constraint and the precision of central bank's signal q. For μ_0 close to either zero or one, prior uncertainty is low, whereas it is high for μ_0 close to a half. A large q means that the research is credible, and effective in guiding agents' beliefs and thus reduce uncertainty.

3.4 Taming Inflation with Communication

Let us start by analyzing efficient economies driven by productivity shocks only (i.e. with $\kappa_1^{\mathcal{M}} = \kappa_1^{\star\mathcal{M}} = 0$ and $\kappa_1^{A} > 0$). This implies that $\mathcal{M}_t = \mathcal{M} = 1$ and therefore the covariance term in equation (Household's welfare) is zero. In this case, the only two terms left are the expected volatility of output gap and inflation.[28] As long as the central bank has access to precise information, and thus firms believe the outcome of the research, there is no tradeoff between stabilizing inflation and output gap.

Proposition 1. *Let $q \to 1$. The optimal communication is fully revealing.*

This result is a direct consequence of theorem (1), which implies that in efficient economies the uncertainty channel is stronger than the coordination channel. In particular, when q is large enough, the central bank can substantially reduce uncertainty that firms have about the realization of the productivity shock. Since the economy is efficient, from an ex-ante perspective, this reduction in uncertainty is optimal.

However, when the research conducted by the central bank is not as credible, uncertainty may increase if its outcome is conflicting with firms' prior belief. The next proposition clarifies the tradeoff that arises in this case in terms of optimal use of information between a communication that targets only inflation versus a policy that targets only the output gap.

Proposition 2. *Let $q \to 1/2$.*

- *Suppose that welfare is given by $\mathbb{E}_{t-1}\,\mathcal{U}_t = -\frac{1}{2}\,\mathbb{E}_{t-1}\,\sigma^2_{\log p_{it}}$. There exists a $\tilde{\mu}_0 \leq 1/2$ such that the optimal communication is non-disclosing for $\mu_0 \in [0, \tilde{\mu}_0] \cup [1-\tilde{\mu}_0, 1]$ and fully revealing for $\mu_0 \in [\tilde{\mu}_0, 1-\tilde{\mu}_0]$.*

[28] For the reasons explained before, in this and following sections I will refer loosely to the cross-sectional dispersion in prices as volatility of inflation.

- *Suppose that welfare is given by* $\mathbb{E}_{t-1}\,\mathcal{U}_t = -\frac{1}{2}\,\mathbb{E}_{t-1}\,\hat{x}_t^2$. *There exists a* $\tilde{\mu}_0 \leq 1/2$ *such that the optimal communication is fully revealing for* $\mu_0 \in [0,\tilde{\mu}_0] \cup [1-\tilde{\mu}_0, 1]$ *and fully revealing for* $\mu_0 \in [\tilde{\mu}_0, 1-\tilde{\mu}_0]$ *if and only if* p *is small enough.*[29]

When communication has no effect on uncertainty, only the coordination channel is present. In this case, there is a potential conflict between stabilizing inflation and stabilizing the output gap. In particular, the most effective way to stabilize expected inflation is to fully reveal available information when uncertainty is low, and not disclose anything when uncertainty is high. On the other hand, as long as firms are not much better informed than the central bank ($p \approx q$), an optimal communication that targets only the output gap is always fully revealing. Therefore, there is always a tradeoff between inflation and output gap when uncertainty is high, but for low uncertainty the tradeoff arises if and only if private knowledge cannot substitute for public information. Since $\alpha^\star - \alpha > 0$, welfare losses from volatility of inflation contribute to a larger extent to welfare than losses from volatility of the output gap, and, for a sufficiently large wedge, an optimal communication prioritizes the stabilization of inflation over the output gap. The next proposition clarifies the condition for inflation targeting to be optimal.

Proposition 3. *There exist a* $\tilde{\mu}_0 \leq 1/2$*, and two thresholds* $\underline{q}$ *and* $\bar{q}$ *with* $\frac{1}{2} < \underline{q} < \bar{q} < 1$ *such that:*

- *For* $\mu_0 \in [0,\tilde{\mu}_0] \cup [1-\tilde{\mu}_0, 1]$*, the optimal communication policy is fully revealing for all* $q > \bar{q}$*. Also, it is non-disclosing for all* $q \leq \bar{q}$ *if and only if*[30]

$$\frac{\omega(\eta-2)}{1+\eta\omega-\eta} > \frac{p(1-p)}{(2p-1)^2} \qquad \text{(Inflation targeting)}$$

- *For* $\mu_0 \in [\tilde{\mu}_0, 1-\tilde{\mu}_0]$*, the optimal communication policy is fully revealing for all* $q \notin (\underline{q}, \bar{q})$*. Also, it is partially revealing for all* $q \in (\underline{q}, \bar{q})$ *only if condition (Inflation targeting) holds.*

For intermediate levels of q, the two channels interact in a non-trivial way. In particular, when prior uncertainty is high and the central bank does not have access to precise information, not disclosing any information is optimal as long as agents already have precise private knowledge (high p), or the wedge $\alpha^\star - \alpha$ is large, meaning that welfare would benefit a lot from stronger pricing complementarities and lower inflation. Condition (Inflation targeting) characterizes how large the wedge has to be in terms of preferences, technology, and precision of the private information for inflation targeting to be optimal.[31] In particular,

[29] The condition is $(\eta-1)(\omega-1)(\eta-\omega\eta-1-\omega)\frac{1}{\omega^2} \leq \frac{1}{2}\frac{p(1-p)-\frac{1}{8}}{p^2(1-p)^2}$. The RHS is decreasing in p and equal to 1 for $p = 1/2$. The LHS is always below 1 and decreasing in ω.

[30] The LHS is increasing in ω if and only if $\eta > 2$, and increasing in η if and only if $\omega > \frac{1}{2}$. The RHS is decreasing in p.

[31] Inflation targeting is here meant in the sense of proposition (2), that is that the optimal policy coincides with the optimal policy of a fictitious economy in which the objective function of the central bank targets only inflation.

for $\eta < 2$ the condition is never met and the optimal policy is fully revealing for all qs. The intuition is that η and ω affect both the equilibrium coordination level and the efficient coordination level: a higher ω always decreases the wedge $\alpha^\star - \alpha$, whereas η increases it if and only if it is lower than two. A large wedge means that the volatility of inflation is inefficiently high, and thus the central bank has a stronger incentive to target it. On the other hand, when prior uncertainty is low, the optimal communication is fully revealing for both high and low precision of the private information available to the central bank, but it can be either fully or partially revealing for intermediate values of q. The intuition for this is that, when q is low, the action of the central bank is very constrained, hence if revealing is optimal then fully revealing the little knowledge available is optimal. For q large, the usual effect on uncertainty kicks in, and full revelation is again optimal. Only at intermediate values of q the communication has enough leeway to generate a tradeoff, but it is not powerful enough to fully resolve uncertainty.

Let us now focus on the latter case. With a partially revealing communication, the central bank tosses a coin upon observing an unfavorable outcome of the research, and thus induces non-fundamental fluctuations of the economy.

Proposition 4. *Suppose that $\mu_0 \in (\frac{1}{2}, 1 - \tilde{\mu}_0)$ and the optimal communication policy is partially revealing. Then, the probability of announcing an expansion conditional on a favorable outcome of the research is $\pi^y(y_t = 1|s_t = 1) = 1$, whereas the probability conditional on an unfavorable outcome of the research is $1 > \pi^y(y_t = 1|s_t = 0) > 0$ and it is increasing in the distance $\mu_0 - \frac{1}{2} > 0$.*

When agents believe that an expansion is likely, the central bank uses information strategically to confirm that belief and influence agents' expectations, even when an unfavorable outcome of the research may increase uncertainty. The optimal way to convince agents of the expansion, while still making sure that the announcement is not disregarded as pure noise, is to randomize between true information content (i.e. announcing a recession) and "lying" (i.e. announcing an expansion) upon getting evidence suggestive of a recession. The probability of lying $\pi^y(y_t = 1|s = 0)$ is increasing in the confidence that agents have of a boom, as summarized by the distance between the prior and a half.

There are here two sources of volatility that are inherited by pricing decisions and thus welfare, and generated from the randomness of the public signal y_t: the first is due to the fundamental uncertainty in the outcome of the research, as summarized by $q < 1$; the second is due to policy-driven uncertainty, coming from the additional noise that is used strategically to persuade firms of the expansion. Hence, sunspot fluctuations can here arise as the result of an optimal communication policy, but in efficient economies this does not happen when $q \to 1$. We will see now that partial revelation is actually a robust feature of inefficient economies for any value of q.

3.5 Optimal Communication with Markup Shocks

Let us now move to the case in which the only source of the business cycle is a countercyclical aggregate markup shock (i.e. $\kappa_1^A = 0$ and $\kappa_1^{\mathcal{M}} > 0$).[32] This introduces a new tradeoff, because now not disclosing information improves welfare through efficiency gains, as captured by the covariance term in equation (Household's welfare), but reduces welfare through volatility of the output gap and inflation.

Proposition 5. *There exists a $\tilde{\mu}_0 \leq 1/2$ such that*

- *For $\mu_0 \in [0, \tilde{\mu}_0] \cup [1 - \tilde{\mu}_0, 1]$, the optimal communication is non-disclosing.*
- *For $\mu_0 \in [\tilde{\mu}_0, 1 - \tilde{\mu}_0]$ and $\eta < 2$,[33] the optimal communication is non-disclosing if and only if*

$$\omega > \frac{(\eta - 1)(4p(1-p))}{1 + (\eta - 1)(-12p^2 + 12p - 2)} \qquad \text{(Markup targeting)}$$

Otherwise, there exists a threshold $\bar{q}$ such that the optimal communication is partially revealing for all $q \geq \bar{q}$ and fully revealing for all $q < \bar{q}$.

In inefficient economies, non-disclosing information is optimal much more often than in efficient economies. This is intuitive, as now there is an additional force that pushes towards non-disclosure. In particular, non-disclosure is always optimal even when communication is mostly powerful ($q \approx 1$) and prior uncertainty is high. This follows from the fact that, when uncertainty is high, volatility of inflation contributes to larger welfare losses than the volatility of the output gap and the best way to tame inflation is a non-disclosing communication. On the other hand, when uncertainty is low to begin with, an optimal communication targets inefficient fluctuations from the markup shock over inflation provided that ω is large enough. Condition (Markup targeting) is equivalent to requiring that firms produce with decreasing returns (i.e. $\omega > 1$) whenever $\eta - 1 > \frac{1}{2(2p-1)^2}$, which happens for example when p is large. The intuition is that a large ω increases profits of the firms and thus efficiency gains.

Finally, partial revelation can occur even in the case of q large, which shows that, in inefficient economies, policy-induced sunspot fluctuations can be optimal even when the central bank has the option to fully resolve uncertainty.

[32] Including both productivity and markup shocks does not change qualitatively the interesting case of intermediate levels of inefficiency as in corollary (5), but would allow to discuss also the two extreme cases with small inefficiency ($\kappa_1 \approx \kappa_1^\star$) from corollary (2) and large inefficiency ($\kappa_1 - \kappa_1^\star$ large) from corollary (3), where $\kappa_1 \equiv \kappa_1^A - \kappa_1^{\mathcal{M}}$ and the efficient covariance of prices with the state given by given by $\kappa_1^\star \equiv \kappa_1^A$. With the assumption of $\kappa_1^A = 0$, the optimal policy does not depend on $\kappa_1^\star - \kappa$.

[33] The assumption of $\eta < 2$, which is consistent with reasonable calibrations of the aggregate markup, implies that condition (Negative at 1/2) is not satisfied for any p. When the condition is satisfied, the optimal policy is partially revealing for $\mu_0 \in [\tilde{\mu}_0, 1 - \tilde{\mu}_0]$. It can be relaxed to any value of η for p small enough. See the proof for details. Also, this is the most interesting case as the difference with proposition (3) is sharpest.

3.6 Some Principles for Central Bank Communication

Let us now summarise the main intuitions from the analysis of central bank communication and discuss the robustness of the findings. Table 1 reports schematically the previous results for the business cycle model discussed.

		High q	Low q
Productivity Shocks	Low uncertainty	FR	ND iff Inflation targeting
	High uncertainty	FR	FR/PR
Markup Shocks	Low uncertainty	ND	ND
	High uncertainty	ND iff Markup targeting	

Table 1: Optimal public communication policy in business-cycle models with strategic complementarities and information frictions. FR stands for fully revealing, ND for non-disclosing, PR for partially revealing.

First, in order to decide on an optimal communication policy, it is essential to understand the source of business-cycle fluctuations. In fact, it is often optimal to provide information about shocks that generate efficient fluctuations, and often optimal to withhold information about shocks that generate inefficiencies. This principle is very general, and in fact in line with numerous comparative statics results in the previous literature.

Second, there is a substantial difference between disclosing information when that is precise and credible versus when it is not. Precise information is always beneficial in efficient economies, imprecise information can sometimes result in more nuanced tradeoffs between volatility and output gap. This result is again very general, though novel in the literature.

Let us now examine the latter point in more detail in the context of the application. The tradeoff between stabilization of inflation or output gap can generally go in either way, because they are affected by information in different ways through the effect on coordination between firms. The coordination channel operates in the model because of the interaction between pricing complementarities and imperfect information, and thus captures at the essence an important element of strategic interactions of firms under incomplete information. The assumption on the type of competition between firms determines the direction of the coordination externality, so it determines which one of the two variables contributes to larger welfare losses. Complementarities à la Dixit-Stiglitz result in inflation being more important. From previous literature[34] we also know that Cournot competition would lead to similar conclusions, whereas Bertrand competition would lead to the output gap being more important. In

[34] See for example Angeletos and Pavan (2007).

appendix D, I provide an alternative microfoundation that can accommodate parsimoniously for all these cases.

This tradeoff results in a condition on preferences, technology, and information such that inflation targeting (as opposed to output gap targeting) is optimal. A novel insight is that this condition can be restrictive or always satisfied depending on firms' prior uncertainty. In models with Dixit-Stiglitz complementarities, inflation targeting may not be optimal only when uncertainty is low. The intuition is that imprecise information can confuse firms and increase unnecessarily uncertainty. On the other hand, when firms are highly uncertain about the state of the economy, central bank communication always helps with taking more accurate pricing decisions.

Inefficient shocks such as markup shocks generate an additional incentive towards not disclosing, as long as deviations from the frictionless benchmark reduce the inefficiency in the economy–e.g. countercyclical markups. When uncertainty is low, stabilizing inflation and generating efficiency gains can be both achieved with a non-disclosing policy, therefore in models with Dixit-Stiglitz complementarities there is no tradeoff. A tradeoff is present when uncertainty is high, and it leads to a condition for markup targeting to be optimal.

Also, previous literature has discussed "self-fulfilling equilibria" that can arise in this type of models with imperfect information, that is equilibria that display ex-post sunspot fluctuations.[35] The analysis of the optimal communication policy shows that it is not obvious that shutting down sunspot fluctuation is necessarily optimal, and in fact it is generally not in inefficient economies. Partial revelation of information, which is in a sense the most "strategic" use of information and induces policy-driven sunspot fluctuations, can be the best way to use information. It is intuitively the case when there is an inefficiency to fix. Perhaps more surprisingly, it can also be the case in efficient economies, when public information is imprecise or not credible and can lead to an increase in uncertainty.

To conclude, all the results above do not depend specifically on the fact that the the rigidity is nominal, and in fact would hold also in a setting with real rigidities arising from imperfect information, as shown in appendix D. The model in the application can be extended in many directions without affecting much the main intuitions, such as it can accommodate for optimal traditional monetary policy, or nominal and real rigidities at the same time.

4 Related Literature

The question of the effects of information on welfare in models with complementarities and strategic uncertainty has been introduced in the seminal works of Clarke (1983) and Morris and Shin (2002), and studied extensively in a large and growing literature. Previous research has focused the attention on comparative statics exercises. Up to my knowledge, this is the first article to study optimal

[35] See for example Benhabib et al. (2015), Benhabib et al. (2016b), and Acharya et al. (2021).

information policies in this class of games. By doing that, this paper merges the beauty-contest literature with that on information design, that developed from the seminal works of Myerson (1986) and Kamenica and Gentzkow (2011), and is currently an active area of research.

The debate around the effects of public information was spurred by influential discussions of Svensson (2006), Morris et al. (2006), Angeletos and Pavan (2007), and James and Lawler (2011) among others. These papers studied incentives and tradeoffs in increasing the precision of a gaussian public signal in beauty-contest games. This article contributes to that discussion by showing that also the assumption of gaussian information structure comes with restrictions, and sometimes loss of generality. In particular, when allowing for a full unrestricted distribution, the cases in which not disclosing information is optimal are further reduced.

A related result on optimal communication policy in that literature can be found in an application from Bergemann and Morris (2013). In the application, the authors consider quantity (Cournot) competition in a market, where the information designer wants to maximize the sum of the firms' payoffs, i.e., the industry profits, and information is incomplete and normally distributed. The planner can send private signals and induce players to make total output choices that are closer to the optimal level, but allow them to negatively correlate their output. Differently from that application, this research focuses the attention on a public signal and relaxes the assumption of normality.

This paper also contributes to the information design literature. In particular, it is mostly related to the work on bayesian persuasion with multiple informed receivers as in Alonso and Câmara (2016), Kolotilin et al. (2017), Bardhi and Guo (2018), Basak and Zhou (2020), and Mathevet et al. (2020). Differently from that research, I focus the attention on a beauty contest model, which has direct applications in the macroeconomic literature, and on a public signal, which is particularly relevant for the central bank application. Also, I show how the optimal policy depends on distortions in the economy (efficiency and coordination externality), as well as the precision of the information available to the sender. Related to the latter, the approach to the solution of sender's problem under imperfect private knowledge of the state that is adopted here applies beyond the setting studied, and can be easily introduced in other frameworks to discuss how optimal information policies change when the sender does not directly observe the state.

In the contest of global games, several papers discussed the optimal design of information in games with multiple receivers, incomplete information, complementarities, higher-order uncertainty, and public signals. Goldstein and Huang (2016) analyze the information design problem in a regime change game, which features coordination among multiple receivers who have heterogeneous private information, and restrict the attention to monotone pass/fail policies. Under commitment, the planner achieves full coordination among agents. Inostroza and Pavan (2021) study adversarial persuasion in global games, where agents are endowed with exogenous private information and the designer is constrained to disclose the same information to all market participants. The authors show that, in that model, the planner induces all agents to take the same action, hence it minimizes dispersion. This paper, on the other hand, studies the trade-off between dispersion, volatility, and efficiency, and rationalizes the possibility of implementing a corner solution with an information aggregation constraints that captures the ability of the planner to collect information that is dispersed, and difficult or costly to aggregate. In particular, it is often not optimal to fully reduce strategic uncertainty, which is an important difference with respect to their global game setting.

In the same spirit of the application of this paper, Angeletos and Sastry (2021) study optimal public communication policies of the monetary authority. There, communication is "Odyssean" forward guidance, i.e. an announcements of future targets or policies; complementary, this paper studies "Delphic" communication, that is announcements that have an information content about fundamentals.[36] The latter channel has been emphasized, among others, in empirical work by Nakamura and Steinsson (2018) and Jarociński and Karadi (2020). Also, Herbert (2021) studies the information design problem of the central bank in a model without the beauty-contest component, which is nested as a special case of this paper. Iovino et al. (2022) extend Angeletos and La'O (2020) to the case in which the central bank is subject to imperfect information about the state. Their comparative statics result can be understood as a special case of the efficient economies studied here, under a restricted gaussian information structure. Consistent with a fully revealing optimal communication, the authors also find that, in an efficient economy, increasing the precision of public information is always beneficial as long as information available to the planner is sufficiently precise. Other related and influential applications to business-cycle models with information frictions are Angeletos and La'O (2013), Benhabib et al. (2015), and Angeletos et al. (2016).

5 Conclusions

This paper studies optimal public communication under commitment in economies featuring strategic uncertainty arising from complementarities and information frictions. These economies are particularly well suited to capture a number of macroeconomic forces arising from interactions in general equilibrium, including sentiments or animal spirits as intended in the work of Keynes (1937). In the spirit of Hayek (1945) and Radner (1962), the potency of public communication is restricted by an information aggregation constraint, which captures the imperfect ability of the planner to collect information that is dispersed, and difficult or costly to aggregate.

The main theoretical results of the paper characterize how the optimal public communication changes depending on the distortions present in the economy, the prior level of uncertainty of the agents in the economy, and the precision of the information (or credibility) available to to the planner. With some caveats,

[36]See Campbell et al. (2012), Andrade and Ferroni (2021), Andrade et al. (2019) and related literature for empirical evidence on the relative importance of these two channels.

the optimal communication tends to reveal information when the economy is efficient, and not disclose information when inefficient. Prior uncertainty of the agents is important, because volatility of the average action and cross-sectional dispersion of actions, which are the key variables for welfare calculations, are affected differently by information depending on whether prior uncertainty is high or low. Also, depending on the distortions of the economy, the planner might want to target preferentially dispersion or volatility. Finally, in efficient economies, precise information is always welfare improving, whereas imprecise information may lead to an increase in agents' uncertainty which offsets the benefits.

These theoretical results are interpreted through the lenses of a business-cycle model with pricing complementarities à la Dixit-Stiglitz and information frictions. In this environment, the paper characterizes the optimal communication policy under commitment. Efficiency and inefficiency of the economy are modelled as productivity and countercyclical markup shocks respectively. Since in the model inflation is excessively volatile compared to the output gap, the planner has an incentive to use communication to preferentially stabilize inflation. When the economy is driven by productivity shocks, and the central bank has precise information about the realization of the shock, there is no tradeoff between stabilization of the two, and full revelation simply achieves approximately the first best. With imprecise information, a condition on preferences, technology, and information is derived for inflation targeting to be optimal. When the economy is driven by countercyclical markup shocks, there is an additional incentive not to disclose information, as it generates efficiency gains. In this case, a strategic partial revelation of information is often the optimal way to use information.

The paper did not touch on important points that may be relevant to understand optimal communication, and can be investigated in follow-up research. First, the setting is static, thus abstracts from dynamic incentives to reveal information, the role of learning, the acquisition of information, and the connection between credibility and commitment. Interesting new lines of research in these directions are the works of Huo and Takayama (2015) and Angeletos and Huo (2021), that highlight how persistence of information translates into persistence of the dynamics of aggregates. Learning, and in particular social learning under model misspecification as in the work of Bohren and Hauser (2021), is an important connection that can be studied in settings displaying strategic uncertainty. Information acquisition is a natural extension that could provide a rationale for imperfect private information of agents and planner, such as in the work of Colombo et al. (2014) and Myatt and Wallace (2014). Also, recent work by Mathevet et al. (2019) proposes a way to relax the assumption of commitment with repeated interactions between sender and receivers. Second, the theoretical framework is ultimately intended to advice for policy, as discussed by means of the illustration; however, the connection between the theoretical results and empirical measurement of central bank communication as in the works of Nakamura and Steinsson (2018) and Jarociński and Karadi (2020) is at best suggestive, and requires further quantitative exploration.

Appendix

Notation for the Appendix. Let $\tilde{\mu}_y(\theta) := \pi^y(y|\theta)\tilde{\mu}_0(\theta)/\big(\sum_{\theta'} \pi^y(y|\theta')\tilde{\mu}_0(\theta')\big)$ be the posterior distribution induced by the public signal computed using Bayes rule, and $\pi^y(y|\theta) = \sum_{s \in S} \pi^y(y|s)\pi^s(s|\theta)$. Let $\mu_0 := \tilde{\mu}_0(1)$ and $\mu_y := \tilde{\mu}_y(1)$ the posterior probabilities that the state is high, respectively, unconditionally and conditionally on the public signal. I will denote expectations with respect to the different distributions without using the tilde to ease notation. In the appendix, actions and aggregates as a function of the public signal are denoted with a hat, and as a function of the posterior belief are denoted without the hat. In the main text, with a slight abuse of notation and for simplicity of the notation, both are denoted without the hat.

A Complements to the main text

A.1 Equilibrium Use of Information

Agents' behavior given information is mechanical. Given their knowledge of π^y, they use Bayes' rule to update their belief from the prior $\tilde{\mu}_0$ to the posterior $\tilde{\mu}_y$, and they select the action k_i which maximizes expected utility given the private information $\mathbb{E}_{\mu_y}(u_i|x_i)$. The problem simplifies because the assumption of i.i.d. realizations of x implies that aggregates K, σ_k depend only on the public signal and the state (θ, y).

The concept of equilibrium used here is the standard Bayes-Nash equilibrium, defined as follows.

Definition 2 (Equilibrium)**.** *A symmetric Bayes-Nash equilibrium is a strategy $\hat{k} : X \times Y \to \mathbb{R}$ such that, for all $(x, y) \in X \times Y$:*

$$\hat{k}(x,y) = \arg\max_{k'} \mathbb{E}_{\theta \sim \tilde{\mu}_0}[U(k', \hat{K}(\theta,y), \hat{\sigma}_k(\theta,y), \theta)|x,y] \tag{1}$$

where

$$\hat{K}(\theta,y) \equiv \sum_x \hat{k}(x,y)\pi^x(x|\theta)$$

and

$$\hat{\sigma}_k(\theta,y) \equiv \left(\sum_x (\hat{k}(x,y) - \hat{K}(\theta,y))^2 \pi^x(x|\theta)\right)^{1/2}$$

for all (θ, y).

Following standard arguments, a direct maximization of utility from equation (1) leads to the first-order condition $\mathbb{E}_{\mu_0}[U_k(\hat{k}, \hat{K}, \hat{\sigma}_k, \theta)|x,y] = 0$. Expanding the first-order condition, it is immediate to show that the unique solution is linear:[37]

$$\hat{k}(x,y) = \mathbb{E}_{\mu_0}[(1-\alpha)\kappa(\theta) + \alpha\hat{K}(\theta,y)|x,y]$$

where $\alpha := U_{kK}/|U_{kk}| \in (-1,1)$ is the *privately optimal level of coordination*.[38] This representation of the solution is a linear combination of two conditional expectations, that of the optimal action under complete information and the expectation of the average action in the population. In practice, solving for the equilibrium amounts to compute those conditional expectations, and imposing a rational expectation fixed point. Given the simple structure of information, this is easy to do. First, notice that the equilibrium strategy $\hat{k}(x,y)$ can be rewritten as $k(x,\mu_y)$, a function of x and μ_y only. This can be done because $\hat{k}$ depends on y only through μ_y.

$$\begin{aligned}\hat{k}(x,y) =& \arg\max_{k'} \mathbb{E}_{\theta\sim\tilde{\mu}_0}[U(k',\hat{K}(\theta,y),\hat{\sigma}_k(\theta,y),\theta)|x,y] \\ =& \arg\max_{k'} \mathbb{E}_{\theta\sim\tilde{\mu}_y}[U(k',\hat{K}(\theta,\mu_y),\hat{\sigma}_k(\theta,\mu_y),\theta)|x] \\ =& k(x,\mu_y)\end{aligned} \tag{2}$$

The solution of the decentralized economy can now be characterized in terms of the posterior induced by the public signal and private information.

Proposition 6 (Characterization of equilibrium)**.** *The unique equilibrium strategy is given by*

$$k(x,\mu_y) = \begin{cases}\kappa_0 + \kappa_1\bar{k}(\mu_y) & x=1\\ \kappa_0 + \kappa_1\underline{k}(\mu_y) & x=0\end{cases}$$

where

$$\bar{k}(\mu_y) := \mathbb{E}_{\mu_y}\{\theta|x=1\} - \gamma(x=1)\left(\frac{\alpha}{1-\alpha}\right)\mathbb{E}_{\mu_y}\{\theta|x=0\}$$

$$\underline{k}(\mu_y) := \mathbb{E}_{\mu_y}\{\theta|x=0\} - \gamma(x=0)\left(\frac{\alpha}{1-\alpha}\right)\mathbb{E}_{\mu_y}\{\theta|x=1\}$$

and

$$\gamma(x) := \frac{\mathbb{E}_{\mu_y}\{\theta|x\} - \mu_y}{\frac{\mu_y^2}{\mathbb{E}_{\mu_y}\{\theta|x\}} + \left(\frac{\alpha}{1-\alpha}\right)\mathbb{E}_{\mu_y}\{\theta|1-x\}}$$

with $\gamma(x=0) \leq 0 \leq \gamma(x=1)$.[39]

[37] Expand U_k around the point $(\kappa,\kappa,0,\theta)$ and rearrange. Moreover, given that $\alpha \in (-1,1)$, one can show along the lines of Morris and Shin (2002) that the linear solution is unique for a bounded action space.

[38] The complete-information allocation κ and the optimal level of coordination α are pinned down by preferences, but one can think of them as primitives of the model, which fully characterize agents' behavior for any information structure, and ex-post retrieve the preferences that led to those parameters. This is often easier to do in applications or when mapping this setting to existing models.

[39] With the convention that $\gamma = 0$ whenever $\mu_y = \mathbb{E}_{\mu_y}\{\theta|x\}$.

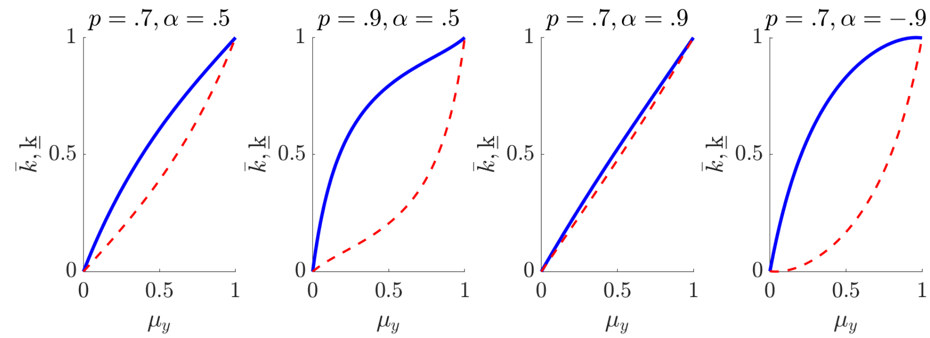

Figure 3: Comparative statics on $\bar{k}(\mu_y)$ (blue) and $\underline{k}(\mu_y)$ (red dashed).

The equilibrium strategy is characterized by two functions, $\bar{k}(\mu_y)$ and $\underline{k}(\mu_y)$, that describe how the best response changes when the public information available to agents changes. The functions are two because each one of them is associated to a possible realization of the private signal. $\bar{k}(\mu_y)$ and $\underline{k}(\mu_y)$ are the sum of two terms: the first one is the bayesian posterior expectation of the state after observing also the private signal; the second is a distortion towards the average action that arises from the coordination motive in equation (Best response). When $\alpha = 0$, the second term vanishes, and the action only reflects the bayesian posterior mean $\mathbb{E}_{\mu_y}\{\theta|x\}$ induced by observing realization x of the private signal. When $\alpha \neq 0$, the best response assigns a positive weight to the posterior mean that would have been induced if the agent had observed the other signal realization $1-x$ of the private signal. As discussed in Huo and Pedroni (2020), the intuition is that the coordination motive associated with $\alpha > 0$ can be thought as a distortion in the precision of the signals received, with a weight for the public signal that is distorted upward and increasing in α. This channel is summarized by the parameter γ, which captures the intensity of the distortion towards the average. $|\gamma|$ is increasing in the distance between the the posterior belief after and before observing the private realization $|\,\mathbb{E}_{\mu_y}\{\theta|x\} - \mu_y|$.

Figure (3) shows comparative statics for the optimal actions. An increase in p generates more dispersion, whereas an increase in α reduces dispersion. In particular, when $\alpha \to 1$ the action $k(x, \mu_y) \to \kappa_0 + \kappa_1 \mu_y$ becomes linear in the posterior induced by the public signal and does not depend on the realization of x. The intuition comes from the fact that, when coordination among agents is maximally relevant for payoffs, agents do not care enough about the fundamentals to put some weight on their private information, and exploit the fact that y is a public signal to coordinate around it. This limiting case clarifies the sense in which the public signal can be used as a coordination device by agents, which is the main difference between private and public information in this setting.

A.2 Efficient Use of Information

Definition 3 (Efficient allocation)**.** *An efficient allocation is a triple* $(\hat{k}^\star, \hat{K}^\star, \hat{\sigma}_k^\star)$ *with* $\hat{k}^\star : X \times Y \to \mathbb{R}$*,* $\hat{K}^\star : \Theta \times Y \to \mathbb{R}$*, and* $\hat{\sigma}_k^\star : \Theta \times Y \to \mathbb{R}_+$ *such that, for almost all* $(\theta, x, y) \in \Theta \times X \times Y$*:*

$$\left(\hat{k}^\star(x,y), \hat{K}^\star(\theta,y), \hat{\sigma}_k^\star(\theta,y)\right) = \underset{k', K', \sigma_k'}{\arg\max}\, \mathbb{E}_{\theta\sim\tilde{\mu}_0}\, \mathbb{E}_{y\sim\pi^y(\theta)}\, \mathbb{E}_{x\sim\pi^x(\theta)}\, U(k', K', \sigma_k', \theta)$$

$$\text{subject to}$$

$$K' = \sum_x k'\pi^x(x|\theta) \quad a.e.\ (\theta, y)$$

$$\sigma_k' = \left(\sum_x (k' - K')^2 \pi^x(x|\theta)\right)^{1/2} \quad a.e.\ (\theta, y)$$

The efficient allocation represents the solution of an hypothetical maximization problem of a constrained planner that is subject to the same information frictions as the agents, but is otherwise free to choose allocations, i.e. the mappings between realization of signals and actions. In this sense, it is a second-best allocation. Similarly to the equilibrium allocation and along the lines of Angeletos and Pavan (2007), one can show that the efficient allocation is a linear combination of the efficient action under complete information $\kappa^\star(\theta)$ and the average action, that is

$$\hat{k}^\star(x,y) = \mathbb{E}_{\mu_0}[(1-\alpha^\star)\kappa^\star(\theta) + \alpha^\star \hat{K}(\theta,y)|x,y]$$

where the weight is the *efficient level of coordination*

$$\alpha^\star := 1 - \frac{U_{kk} + 2U_{kK} + U_{KK}}{U_{KK} + U_{\sigma\sigma}} \in (-1, 1)$$

Also, $\kappa^\star(\theta) = \kappa_0^\star + \kappa_1^\star\theta$ is the unique solution[40] to $W_k(\kappa^\star, 0, \theta) = 0$, where

$$W(K, \sigma_k, \theta) := \int_0^1 U(k_i, K, \sigma_k, \theta)di = U(K, K, \sigma_k, \theta) + \frac{1}{2}U_{kk}\sigma_k^2 \tag{3}$$

denotes welfare under the utilitarian aggregator. This leads to the characterization of the efficient allocation, which is equivalent to proposition (6), once replaced α with $\alpha^\star$ and $\kappa(\theta)$ with $\kappa^\star(\theta)$.

Proposition 7 (Characterization of the efficient allocation)**.** *The unique efficient allocation is given by* $(k^\star(x,\mu_y), K^\star(\theta,\mu_y), \sigma_k^\star(\theta,\mu_y))$ *with*

$$k^\star(x,\mu) = \begin{cases} \kappa_0^\star + \kappa_1^\star \bar{k}^\star(\mu) & x = 1 \\ \kappa_0^\star + \kappa_1^\star \underline{k}^\star(\mu) & x = 0 \end{cases}$$

[40] With $\kappa_0^\star \equiv -\frac{U_k + U_K}{U_{kk} + 2U_{kK} + U_{KK}}$ evaluated at $(0,0,0,0)$, and $\kappa_1^\star \equiv -\frac{U_{k\theta} + U_{K\theta}}{U_{kk} + 2U_{kK} + U_{KK}}$.

where

$$\bar{k}^\star(\mu) \equiv \mathbb{E}_\mu\{\theta|x=1\} - \gamma^\star(x=1)\left(\frac{\alpha^\star}{1-\alpha^\star}\right)\mathbb{E}_\mu\{\theta|x=0\}$$

$$\underline{k}^\star(\mu) \equiv \mathbb{E}_\mu\{\theta|x=0\} - \gamma^\star(x=0)\left(\frac{\alpha^\star}{1-\alpha^\star}\right)\mathbb{E}_\mu\{\theta|x=1\}$$

and

$$\gamma^\star(x) \equiv \frac{\mathbb{E}_\mu\{\theta|x\} - \mu}{\frac{\mu^2}{\mathbb{E}_\mu\{\theta|x\}} + \left(\frac{\alpha^\star}{1-\alpha^\star}\right)\mathbb{E}_\mu\{\theta|1-x\}}$$

with $\gamma^\star(x=0) \leq 0 \leq \gamma^\star(x=1)$.[41]

Proof. The proof for $k^\star(x,\mu)$ follows along the same lines of proposition 6. □

A.3 Optimal Communication in Efficient Economies

In efficient economies, welfare can be rewritten as the sum of two terms. The first term, $\mathbb{E}[(K-\kappa)^2]$, is the volatility of the aggregate action around fundamentals.[42] It arises because of the interaction between complementarities and information frictions generates a wedge between the average action and the allocation under complete information, that is imperfect information does not "average out." The second term, $\mathbb{E}[(k-K)^2]$, is the cross-sectional dispersion of individual actions around the average action. It arises because private information generates heterogeneity across agents.

Lemma 2 (Welfare in Efficient Economies)**.** *In economies with $\kappa = \kappa^\star$, welfare can be rewritten as*

$$\mathbb{E}\,u = \bar{u} + \frac{W_{\sigma\sigma}}{2}\Big\{(1-\alpha^\star)\underbrace{\left(\mathbb{E}[(K-\kappa)^2]\right)}_{Volatility} + \underbrace{\mathbb{E}[(k-K)^2]}_{Dispersion}\Big\} \tag{4}$$

where $\bar{u}$ is constant across all Bayes-plausible τs and $W_{\sigma\sigma} < 0$ is a constant.

This lemma is useful to understand forces at play. An increase in the coordination level α increases volatility and decreases dispersion. Previous literature highlighted that public information can sometimes have a similar effect, because it fosters coordination. In those cases, there is a tradeoff between volatility and dispersion. This tradeoff is the reason for public information to be potentially harmful, because when volatility contributes to larger welfare losses than dispersion ($1-\alpha^\star$ is large), welfare can decrease with the precision of the public signal. In the setting studied here, these forces can be precisely summarized by looking at concavity/convexity patters of volatility and dispersion as a function of the posterior induced by the public signal. Since $W_{\sigma\sigma} < 0$, when volatility

[41] With the convention that $\gamma^\star = 0$ whenever $\mu = \mathbb{E}_\mu\{\theta|x\}$.

[42] In the literature, depending on the application, it is often referred to as non-fundamental volatility, or volatility of the output gap.

or dispersion are concave, information is welfare improving. When one of the two terms is convex,[43] information may be harmful if the relative weight of the convex term is large.

Lemma 3 (Volatility and Dispersion)**.** *There exist intervals (possibly different for volatility and dispersion)* $[0,\mu^\ell]$, $[\mu^\ell,\mu^h]$, *and* $[\mu^h,1]$ *with* $0<\mu^\ell<\frac{1}{2}<\mu^h<1$ *and* $\mu^\ell=1-\mu^h$, *such that:*

- *Volatility is concave in* μ_y *over* $[0,\mu^\ell]\cup[\mu^h,1]$ *and convex over* $[\mu^\ell,\mu^h]$ *if and only if*

$$\frac{\alpha(\alpha-2)}{\alpha(\alpha-2)+1}\geq\frac{1}{2}\frac{p(1-p)-\frac{1}{8}}{p^2(1-p)^2} \tag{5}$$

- *Dispersion is convex in* μ_y *over* $[0,\mu^\ell]\cup[\mu^h,1]$ *and concave over* $[\mu^\ell,\mu^h]$.

The lemma shows that dispersion and volatility are affected by information in different ways. When the prior belief μ_0 is sufficiently close to zero or one, that is agents hold little prior uncertainty about fundamentals, an informative public communication increases welfare losses due to dispersion but decreases losses from volatility. On the other hand, for μ sufficiently close to a half, a tradeoff between volatility and dispersion arises only in the case in which condition (5) is met. In particular, since the LHS is decreasing in α and the RHS is decreasing to $-\infty$ for $p\to1$, the condition holds for p large and α small. This means that when agents are uncertain about fundamentals, precise private information in the absence of strong complementarities leads to public information increasing volatility and decreasing dispersion. For a sufficiently imprecise private signal, the condition is never met and there is no tradeoff between volatility and dispersion as long as agents have a prior belief close to a half, that is public information is unambiguously welfare improving.

A.4 Optimal Communication in Inefficient Economies

Welfare can be rewritten as the sum of three terms, two of which are volatility and dispersion. The novel term is the covariance between the gap (output gap in the application) and the efficiency wedge: when the covariance is positive, it means that a larger gap generates efficiency gains which partially offset losses from volatility.

Lemma 4 (Welfare in Inefficient Economies)**.** *The expected utility of the planner induced by the public signal can be rewritten as*

$$\mathbb{E}\,u=\bar{u}+\frac{W_{\sigma\sigma}}{2}\Big\{(1-\alpha^\star)\Big(\mathbb{E}[(K-\kappa)^2]-2\mathrm{Cov}(\underbrace{K-\kappa}_{\textit{Gap}},\underbrace{\kappa^\star-\kappa}_{\substack{\textit{Inefficiency}\\\textit{Wedge}}})\Big)+\mathbb{E}[(k-K)^2]\Big\} \tag{6}$$

[43]The logic for why concavity/convexity in the μ_y-space is related to the effect of information is that an informative signal, from an ex-ante perspective, induces a randomization between two posteriors. When welfare is convex, randomizing between two posteriors is beneficial by Jensen's inequality.

where $\bar{u}$ is constant across all Bayes-plausible τs and $W_{\sigma\sigma} < 0$ is a constant.

The following corollaries break theorem (3) into cases and discuss implementation and intuitions.

Corollary 1. *Let $q = 1$. In economies with $\kappa = \kappa^\star$, the optimal communication policy is fully revealing, that is*

$$\mu_y = \begin{cases} 1 & \text{if } y = 1 \\ 0 & \text{if } y = 0 \end{cases} \quad \tau = \begin{cases} \mu_0 & \text{if } \mu_y = 1 \\ 1 - \mu_0 & \text{if } \mu_y = 0 \end{cases}$$

and $\pi^y(y = 1|\theta = 1) = \pi^y(y = 0|\theta = 0) = 1$.

This corollary is a special case of theorem (1), and highlights that efficient economies are indeed an particular example of inefficient ones. Conditions (Convexity at $\{0, 1\}$) and (Convexity at $1/2$) on convexity do not need to hold, as evident from figure (1). Nevertheless, welfare is always below zero and the optimal policy is fully revealing. This follows from the fact that conditions (Decreasing at 0) and (Negative at 1/2) hold whenever $\kappa = \kappa^\star$.

Corollary 2. *Let $q = 1$. When $\kappa_1 > 0$,*[44] *there exists a $\bar{\kappa}_1^\star(p, \alpha, \kappa_1, \alpha^\star)$ such that for all $\kappa_1^\star \geq \bar{\kappa}_1^\star$, the optimal communication policy is fully revealing. Moreover, $\bar{\kappa}_1^\star(p, \alpha, \kappa_1, \alpha^\star)$ is increasing in p, κ_1, $\alpha^\star$, and increasing in α if and only if $\alpha - \alpha^\star$ is small enough.*

The corollary shows that, as long as the inefficiency is not too large, the optimal communication policy is fully revealing even in inefficient economies. Hence, in terms of the optimal use of information, an inefficient economy with a small inefficiency is equivalent to an efficient one. Figure (4) shows a graphical representation of welfare and the split into the three components from lemma (4). The red dashed line is the level of welfare that can be achieved for different priors under the optimal policy, the red dots correspond to the support of τ. An optimal communication achieves a constant welfare level for all realizations of the signal. Under this calibration, the covariance term is concave and positive, which means that there is a tradeoff between efficiency gains from less information and volatility. However, distortions are not large enough and losses from volatility dominate. Hence, full revelation is optimal.

The second part of corollary (2) describes how the lower bound on the correlation $\kappa_1^\star$ between the efficient action and fundamental under complete information changes as the other structural parameters change. Intuitively, it is increasing in κ_1, because what matters is the misalignment between planner and agents, rather the level of correlation itself. The threshold also increases in the precision of the private signal, as more private information reduce the wedge $K - \kappa$. Also, it increases in the efficient level of coordination, because a higher $\alpha^\star$ reduces the utility weight assigned to volatility (which dominates the

[44]For $\kappa_1 < 0$, the result applies for $\kappa_1^\star \leq \bar{\kappa}_1^\star(p, \alpha, \kappa_1, \alpha^\star)$. All the next statements will be for the case in which $\kappa_1 > 0$ as well, generalizations to the case $\kappa_1 < 0$ can be easily done. The case $\kappa_1 = 0$ is ruled out by assumptions on primitives because it is uninteresting.

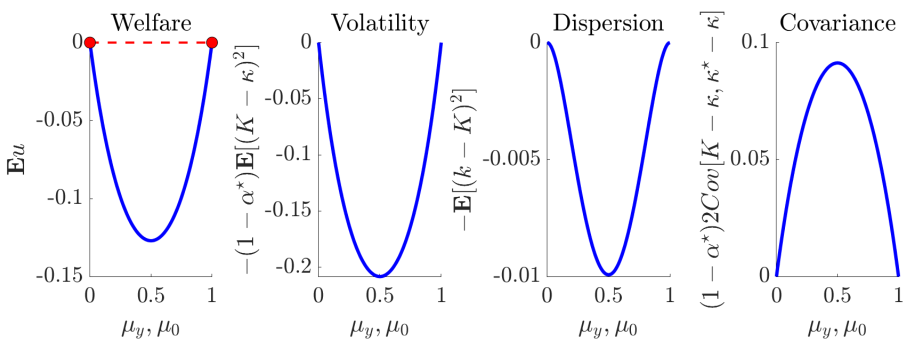

Figure 4: Example of optimal communication policy for corollary (2). Calibration is $p = .7$, $\kappa_0 = 0$, $\alpha = .5$, $\kappa_1 = 1$, $\alpha^\star = 0$, and $\kappa_1^\star = .8$. The optimal communication policy is in red.

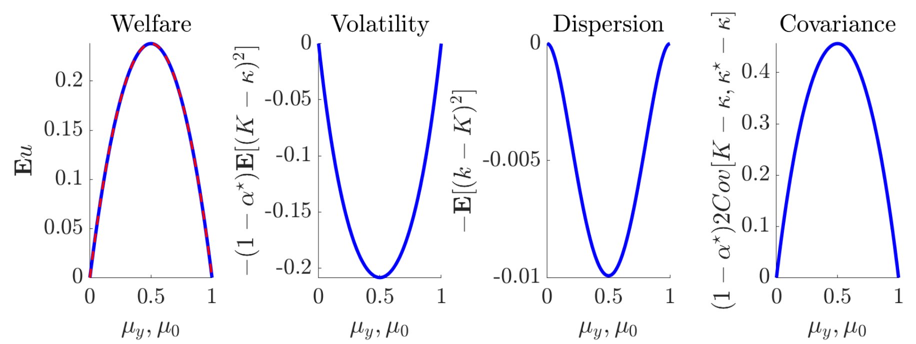

Figure 5: Example of optimal communication policy for corollary (3). Calibration is $p = .7$, $\kappa_0 = 0$, $\alpha = .5$, $\kappa_1 = 1$, $\alpha^\star = 0$, and $\kappa_1^\star = 0$. The optimal communication policy is in red.

covariance term here). Finally, the effect of a change in the level of coordination α is ambiguous as it both reduces dispersion and increases volatility.

Corollary 3. *Let $q = 1$. When $\kappa_1 > 0$, there exists a $\underline{\kappa}_1^\star(p, \alpha, \kappa_1, \alpha^\star)$ such that for all $\kappa_1^\star \leq \underline{\kappa}_1^\star$, the optimal communication policy is non-disclosing, that is $\mu_y = \mu_0$ for all $y \in Y$ and $\pi^y(y = 1|\theta = 1) = \pi^y(y = 0|\theta = 0) = \frac{1}{2}$. Moreover, $\underline{\kappa}_1^\star(p, \alpha, \kappa_1, \alpha^\star)$ is decreasing in p, decreasing in α if and only if $\alpha - \alpha^\star$ is small enough, and increasing in in κ_1 if and only if $\alpha - \alpha^\star$ is small enough, and increasing in $\alpha^\star$.*

Corollary (3) shows that when the inefficiency is large, the planner is willing to suffer losses in terms of dispersion and volatility in order to correct deviations from the first-best allocation. Figure (5) plots welfare for a parametrization consistent with corollary (3). The main difference with respect to the previous figure is that the covariance term is now much larger, showing that first-order losses due to the inefficiency are large and efficiency gains can substantially improve welfare. The planner chooses optimally to tolerate a large volatility of the wedge $K - \kappa$ and dispersion, because it brings the economy closer to the first best. This implies that welfare is concave, and the optimal communication policy is to send a completely uninformative signal, pure noise. Since agents observe the signal structure, the public signal is not used to update expectations and the posterior belief equals the prior belief.

As before, one can gain more intuition by looking at the upper bound on the correlation between the efficient action and fundamentals under complete information. A higher κ_1 increases the threshold, because only the difference $\kappa_1^\star - \kappa_1$ matters for the optimal policy. More precise private information decreases the threshold, because it reduces the the wedge $K - \kappa$; with a smaller covariance, which here dominates dispersion and volatility, inefficiencies must be larger for non-disclosing to be still optimal. Also, a stronger coordination motive among agents increases volatility, which the planner could reduce by disclosing information: the inefficiency must be large enough compared to the coordination externality for non-disclosing to remain optimal.

A strategic use of information comes back into play when the economy is in the intermediate cases. The next two corollaries will show that, when inefficiencies are not too large nor small, what matters for the determination of the optimal communication are the coordination externality and prior uncertainty, as captured by prior beliefs. These provide a characterization of the optimal policy for every prior belief, now distinguishing between a low level of uncertainty with μ_0 close to either zero or one, and high level of uncertainty with μ_0 close to a half.

Corollary 4. *Let $q = 1$. When $\kappa_1 > 0$, condition (Negative at 1/2) does not hold, and $\underline{\kappa}_1^\star \leq \kappa_1^\star \leq \bar{\kappa}_1^\star$, if $\alpha - \alpha^\star$ is large enough, there exist intervals $[0, \mu^\ell]$, $[\mu^\ell, \mu^h]$, and $[\mu^h, 1]$ with $0 < \mu^\ell < \frac{1}{2} < \mu^h < 1$ and $\mu^\ell = 1 - \mu^h$, such that:*

- *for $\mu_0 \in [0, \mu^\ell]$, the optimal communication policy is partially revealing, that is*
$$\mu_y = \begin{cases} \mu_\ell & \text{if } y = 1 \\ 0 & \text{if } y = 0 \end{cases} \quad \tau = \begin{cases} \frac{\mu_0}{\mu^\ell} & \text{if } \mu_y = \mu_\ell \\ 1 - \frac{\mu_0}{\mu^\ell} & \text{if } \mu_y = 0 \end{cases}$$
and $\pi^y(y = 1|\theta = 1) = 1$, $\pi^y(y = 0|\theta = 0) = \frac{\mu^\ell - \mu_0}{\mu^\ell(1-\mu_0)} < 1$.

- *for $\mu_0 \in [\mu^\ell, \mu^h]$, the optimal communication policy is non-disclosing, that is $\mu_y = \mu_0$ for all $y \in Y$ and $\pi^y(y = 1|\theta = 1) = \pi^y(y = 0|\theta = 0) = \frac{1}{2}$.*

- *for $\mu_0 \in [\mu^h, 1]$, the optimal communication policy is partially revealing, that is*
$$\mu_y = \begin{cases} 1 & \text{if } y = 1 \\ \mu^h & \text{if } y = 0 \end{cases} \quad \tau = \begin{cases} \frac{\mu_0 - \mu^h}{1-\mu^h} & \text{if } \mu_y = 1 \\ \frac{1-\mu_0}{1-\mu^h} & \text{if } \mu_y = \mu^h \end{cases}$$
and $\pi^y(y = 1|\theta = 1) = \frac{\mu_0 - \mu^h}{\mu_0(1-\mu^h)} < 1$, $\pi^y(y = 0|\theta = 0) = 1$.

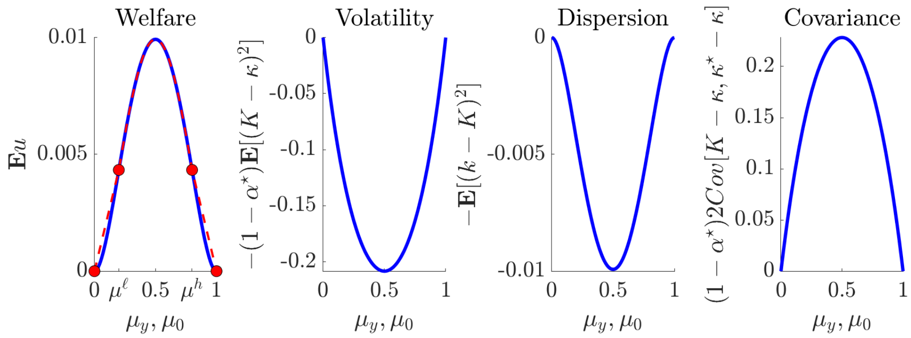

Figure 6: Example of optimal communication policy for corollary (4). Calibration is $p = .7$, $\kappa_0 = 0$, $\alpha = .5$, $\kappa_1 = 1$, $\alpha^\star = 0$, and $\kappa_1^\star = .5$. The optimal communication policy is in red.

The logic for the result is that, at an intermediate level of the inefficiency, there are two regions of the prior beliefs such that volatility losses dominate efficiency gains (low uncertainty), and another in which gains outweigh losses (high uncertainty). With a partially revealing signal, the planner still communicates that the state is high (low) upon receiving indications that it is low (high) when prior beliefs are close to zero (one), with strictly positive probability. On the other hand, upon receiving evidence in favor of the high (low) state, that information is fully revealed. This probability of revealing opposite information than what inferred from the evidence can be chosen such that, in expectations, volatility and efficiency are both reduced. This is optimal up to the point in which the marginal welfare gains from the reduction in volatility equal the marginal losses from the reduction in efficiency. That pins down the probability of such a signal realization, and thus the level of uncertainty–posterior belief–that realizes upon receiving it. Passed that point, further persuasion lowers welfare. Finally, when prior uncertainty is high to begin with, that is prior beliefs are close to a half, efficiency gains are largest and non-disclosure is optimal.

On the other hand, when $\alpha - \alpha^\star$ is small, the relative weight on covariance and volatility is low, and dispersion becomes more relevant in terms of welfare losses. This case is described in the next corollary.

Corollary 5. *Let $q = 1$. When $\kappa_1 > 0$, condition (Decreasing at 0) does not hold, and $\underline{\kappa}_1^\star \leq \kappa_1^\star \leq \bar{\kappa}_1^\star$, if $\alpha - \alpha^\star$ is small enough, there exist intervals $[0, \mu^\ell]$, $[\mu^\ell, \mu^h]$, and $[\mu^h, 1]$ with $0 < \mu^\ell < \frac{1}{2} < \mu^h < 1$ and $\mu^\ell = 1 - \mu^h$, such that:*

- *for $\mu_0 \in [0, \mu^\ell] \cup [\mu^h, 1]$, the optimal communication policy is non-disclosing, that is $\mu_y = \mu_0$ for all $y \in Y$ and $\pi^y(y = 1|\theta = 1) = \pi^y(y = 0|\theta = 0) = \frac{1}{2}$.*
- *for $\mu_0 \in [\mu^\ell, \mu^h]$, the optimal communication policy is partially revealing, that is*

$$\mu_y = \begin{cases} \mu^h & \text{if } y = 1 \\ \mu^\ell & \text{if } y = 0 \end{cases} \quad \tau = \begin{cases} \frac{\mu^h - \mu_0}{\mu^h - \mu^\ell} & \text{if } \mu_y = \mu^h \\ \frac{\mu_0 - \mu^\ell}{\mu^h - \mu^\ell} & \text{if } \mu_y = \mu^\ell \end{cases}$$

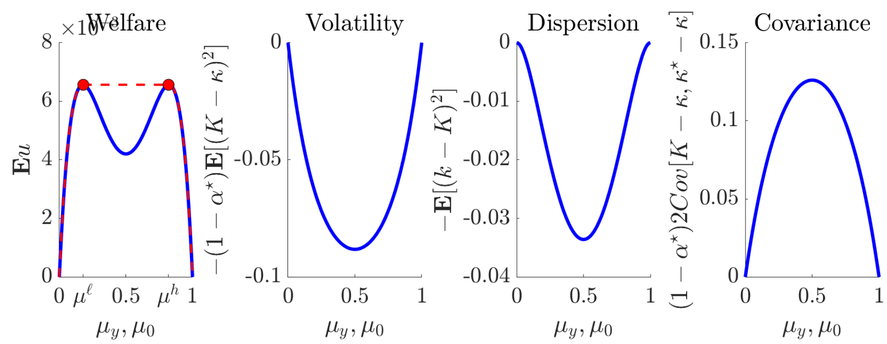

Figure 7: Example of optimal communication policy for corollary (5). Calibration is $p = .7$, $\kappa_0 = 0$, $\alpha = 0$, $\kappa_1 = 1$, $\alpha^\star = .4$, and $\kappa_1^\star = .5$. The optimal communication policy is in red.

$$\pi^y(y=1|\theta=1) = \frac{\mu^h(\mu_0-\mu^\ell)}{\mu_0(\mu^h-\mu^\ell)} < 1,\ \pi^y(y=0|\theta=0) = \frac{(1-\mu^\ell)(\mu^h-\mu_0)}{(1-\mu_0)(\mu^h-\mu^\ell)} < 1.$$

The optimal policy can be visualized in figure (7). When coordination is inefficiently low, dispersion matters more than volatility. As shown in lemma (3), differently from volatility, dispersion increases in public information when it is low (prior close to zero or one), and decreases in information when it is high. Hence, when dispersion and efficiency gains move in the same direction as a function of information, a non-disclosing policy is optimal. For a prior close to a half, there is a tradeoff between dispersion and efficiency, and at this parametrization dispersion dominates. The intuition is that, by revealing only partially the information available, the planner can select the level of posterior uncertainty at which the marginal welfare gain from reducing dispersion equals the marginal cost of reducing efficiency. Above that level, efficiency gains dominate and a non-disclosing policy is optimal.

B Illustration: Model with Nominal Rigidities

B.1 Microfoundation of Aggregate Markup Shock

This section extends the baseline model in the main text to allow for a proper treatment of the markup shock. The modifications are as follows.

The economy consists of a "mainland" and a continuum of "islands." Each island is inhabited by a continuum of workers and a continuum of monopolistically competitive firms. Inputs produced in islands are aggregated in intermediate goods, intermediate goods are aggregated in a final good. The final good is sold in the mainland and consumed by a representative household, which collects all the labor income from local workers and owns all the firms.

Information is share within islands, but can possibly differ across islands. This implies that firms within islands are all identical, and firms across islands differ only because of the different information they have access to.

The final good C_t is aggregated using a CES technology from island-specific intermediate goods c_{it}, where i denotes an island. Intermediate goods are obtained by aggregating inputs c_{ijt}, produced locally by firm j in island i. I assume a standard nested CES structure with elasticity of substitution η across islands and ρ_t within islands.

$$C_t = \left[\int_I c_{it}^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}, \quad c_{it} = \left[\int_J c_{ijt}^{\frac{\rho_t-1}{\rho_t}} dj\right]^{\frac{\rho_t}{\rho_t-1}}$$

where $\eta, \rho_t > 1$ for all t. The elasticity within islands is allowed to change over time, and this generates countercyclical fluctuations in the aggregate markup, common across islands. Standard profit maximization lead to the downward-sloping demand curve faced by firm j in island i:

$$c_{it} = \left(\frac{p_{it}}{P_t}\right)^{-\eta} C_t, \quad c_{ijt} = \left(\frac{p_{ijt}}{p_{it}}\right)^{-\rho_t} c_{it}$$

Firm j in island i produces the differentiated good c_{ijt} using technology $c_{ijt} = A_t n_{ijt}^{1/\omega}$, where n_{ijt} is the local labor demand and ω can accommodate increasing, decreasing, or constant returns to scale. The problem of the firm is $\max\limits_{p_{ijt}, c_{ijt}} p_{ijt}c_{ijt} - W_t n_{ijt}$

Replacing the demand curve into the problem of the firm leads to

$$\max_{p_{ijt}} \mathbb{E}_{it}\left[p_{ijt}\left(\frac{p_{ijt}}{p_{it}}\right)^{-\rho_t}\left(\frac{p_{it}}{P_t}\right)^{-\eta} C_t - W_t\left(\frac{p_{ijt}}{p_{it}}\right)^{-\rho_t\omega}\left(\frac{p_{it}}{P_t}\right)^{-\eta\omega}\left(\frac{C_t}{A_t}\right)^{\omega}\right]$$

The corresponding first-order condition is

$$\mathbb{E}_{it}\left\{(1-\rho_t)\left(\frac{p_{ijt}}{p_{it}}\right)^{-\rho_t}\left(\frac{p_{it}}{P_t}\right)^{-\eta} C_t - (\rho_t\omega)W_t\left(\frac{p_{ijt}}{p_{it}}\right)^{-\rho_t\omega}\frac{1}{p_{ijt}}\left(\frac{p_{it}}{P_t}\right)^{-\eta\omega}\left(\frac{C_t}{A_t}\right)^{\omega}\right\} = 0$$

This implies $p_{ijt} = p_{it}$ for all is. Therefore, replacing the household's first order condition and the cash-in-hand constraint (same as in section B.2), this leads to equation (Firm's FOC), where $\mathcal{M}_t = \left(\frac{\rho_t}{\rho_t-1}\right)\frac{1}{1-\tau}$.

B.2 Derivations for the Main Text

Derivation of Beauty Contest. Replacing the demand curve into the problem of the firm leads to

$$\max_{p_{it}} \mathbb{E}_{it}\left[p_{it}^{1-\eta}P_t^{\eta}C_t - W_t p_{it}^{-\eta\omega}P_t^{\eta\omega}A_t^{-\omega}C_t^{\omega}\right]$$

The corresponding first-order condition is

$$\mathbb{E}_{it}\left[(1-\eta)\left(\frac{p_{it}}{P_t}\right)^{-\eta}C_t + (\eta\omega)W_t\left(\frac{p_{it}}{P_t}\right)^{-\eta\omega}\frac{1}{p_{it}}\left(\frac{C_t}{A_t}\right)^{\omega}\right] = 0$$

The labor supply condition from household's problem is

$$P_tC_t = W_t(1-\tau)$$

Replacing the cash-in-hand constraint into previous equations and rearranging

$$p_{it}^{1+\eta\omega-\eta} = \frac{\mathbb{E}_{it}\left[\omega\mathcal{M}_t M^{\omega}P_t^{\omega(\eta-1)}A_t^{-\omega}\right]}{\mathbb{E}_{it}\left[P_t^{\eta-1}\right]}$$

where the markup $\mathcal{M}_t$ can be microfounded as in section B.1.

Let us now derive the complete-information allocation, which will be denoted with a tilde. In information is complete, one can drop the expectation and solve for $\tilde{p}_{it}$ and $\tilde{P}_t$. In this case all firms will charge the same price and produce the same quantities:

$$\begin{aligned}
\tilde{p}_{it} = \tilde{P}_t &= (\omega\mathcal{M}_t)^{\frac{1}{\omega}}\frac{M}{A_t} \\
\tilde{c}_{it} = \tilde{C}_t &= \left(\frac{1}{\omega\mathcal{M}_t}\right)^{\frac{1}{\omega}}A_t \\
\tilde{n}_{it} = \tilde{N}_t &= \frac{1}{\omega\mathcal{M}_t}
\end{aligned}$$

Using the assumptions for the functional forms of markup and productivity

$$\begin{aligned}
\log\tilde{p}(0) = \log\tilde{P}(0) &= \log M - \frac{1}{\omega}\kappa_0^{\mathcal{M}} - \kappa_0^A \\
\log\tilde{p}(1) = \log\tilde{P}(1) &= \log M - \frac{1}{\omega}(\kappa_0^{\mathcal{M}} + \kappa_1^{\mathcal{M}}) - (\kappa_0^A + \kappa_1^A)
\end{aligned}$$

Log-linearizing equation (Firm's FOC) around the complete-information allocation evaluated at $\theta_t = 0$:

$$\begin{aligned}
(1+\eta\omega-\eta)(\log p_{it} - \log\tilde{p}(0)) \approx\ & \mathbb{E}_{it}\left(\log(\omega\mathcal{M}_t) + \kappa_0^{\mathcal{M}}\right) - \omega\,\mathbb{E}_{it}\left(\log A_t - \kappa_0^A\right) \\
& + (\omega-1)(\eta-1)\,\mathbb{E}_{it}\left(\log P_t - \log\tilde{P}(0)\right)
\end{aligned}$$

Hence, up to a first-order approximation

$$\begin{aligned}\log p_{it} - \log \tilde{p}(0) =& (1-\alpha)\,\mathbb{E}_{it}\left\{\frac{1}{\omega}\left(\log(\omega\mathcal{M}_t) + \kappa_0^{\mathcal{M}}\right) - \left(\log A_t - \kappa_0^A\right)\right\} + \alpha\,\mathbb{E}_{it}\left(\log P_t - \log \tilde{P}(0)\right)\\ \Rightarrow \log p_{it} =& (1-\alpha)\,\mathbb{E}_{it}\left\{\log M + \frac{1}{\omega}\log(\omega\mathcal{M}_t) - \log A_t\right\} + \alpha\,\mathbb{E}_{it}\log P_t\end{aligned} \tag{7}$$

where $\alpha := \frac{(\omega-1)(\eta-1)}{1+\eta\omega-\eta}$ is the equilibrium level of coordination.

Derivation of Welfare. Let us now derive welfare in this economy. First, using the production function, the demand curve, and integrating over is with the market clearing condition $\int n_{it} di = N_t$, we get

$$N_t = \left(\frac{C_t}{A_t}\right)^{\omega} \int \left(\frac{p_{it}}{P_t}\right)^{-\eta\omega} di$$

Define $\hat{x}_t := \log C_t - \log \tilde{C}_t$ to be the output gap, and similar "hat" notation for other variables in log-deviations from the complete-information allocation. Using the fact that $\tilde{p}_{it} = \tilde{P}_t$, taking logs the above implies

$$\frac{1}{\omega}\hat{N}_t = \hat{x}_t + d_t$$

where $d_t := \frac{1}{\omega}\log\int\left(\frac{p_{it}}{P_t}\right)^{-\eta\omega} di$. Expand now up to second order the price index around the complete-information allocation

$$\begin{aligned}\left(\frac{p_{it}}{P_t}\right)^{1-\eta} &= \exp[(1-\eta)(\log p_{it} - \log P_t)]\\ &\approx 1 + (1-\eta)(\log p_{it} - \log P_t) + \frac{(1-\eta)^2}{2}(\log p_{it} - \log P_t)^2\end{aligned}$$

Using $1 = \int\left(\frac{p_{it}}{P_t}\right)^{1-\eta} di$, up to a second-order approximation

$$\int(\log p_{it} - \log P_t)di = \frac{1-\eta}{2}\int(\log p_{it} - \log P_t)^2 di$$

Also, up to a second-order expansion

$$\left(\frac{p_{it}}{P_t}\right)^{-\eta\omega} = 1 - \eta\omega(\log p_{it} - \log P_t) + \frac{(\eta\omega)^2}{2}(\log p_{it} - \log P_t)^2$$

Combining the results

$$\int\left(\frac{p_{it}}{P_t}\right)^{-\eta\omega} di = 1 + \frac{1}{2}\eta\omega(1-\eta+\eta\omega)\sigma^2_{\log p_{it}}$$

because up to second order $\int(\log p_{it} - \log P_t)^2 di = \int(\log p_{it} - \int p_{it} di)^2 di$. Replace into the definition of d_t

$$d_t \approx \frac{1}{2}\eta(1-\eta+\eta\omega)\sigma^2_{\log p_{it}}$$

Expand now static utility $U_t \equiv \log C_t - N_t$ of the household up to a second-order approximation around the complete-information allocation

$$\begin{aligned} U_t - \tilde{U}_t \approx& \frac{1}{\tilde{C}_t}\tilde{C}_t\left(\hat{x}_t + \frac{1}{2}\hat{x}_t^2\right) + \frac{1}{2}\left(-\frac{1}{\tilde{C}_t^2}\tilde{C}_t^2\right)\hat{x}_t^2 - \tilde{N}_t\left(\hat{N}_t + \frac{1}{2}\hat{N}_t^2\right) \\ =& \hat{x}_t - \tilde{N}_t\left(\hat{N}_t + \frac{1}{2}\hat{N}_t^2\right) \end{aligned}$$

Using the fact that $\tilde{N}_t = \frac{1}{\omega\mathcal{M}_t}$ and substituting for $\hat{N}_t$

$$U_t - \tilde{U}_t \approx \hat{x}_t - \frac{1}{\mathcal{M}_t}\left(\hat{x}_t + \frac{1}{2}\eta(1-\eta+\eta\omega)\sigma^2_{\log p_{it}} + \frac{1}{2}\omega\hat{x}_t^2\right)$$

Replace the cash-in-hand constraint $\hat{P}_t = -\hat{x}_t$, $\hat{P}_t^2 = \hat{x}_t^2$

$$U_t - \tilde{U}_t \approx -\left(1 - \frac{1}{\mathcal{M}_t}\right)\hat{P}_t - \frac{1}{2}\frac{\omega}{\mathcal{M}_t}\hat{P}_t^2 - \frac{1}{2}\frac{1}{\mathcal{M}_t}\eta(1-\eta+\eta\omega)\sigma^2_{\log p_{it}}$$

If $\kappa_1^{\mathcal{M}} = 0$ (no aggregate markup fluctuations), the economy is efficient and we have that $\mathcal{M}_t = \mathcal{M} = 1$. This follows from the optimal fiscal subsidy $\log(1+\tau) = -\kappa_0^{\mathcal{M}}$. Hence

$$U_t - \tilde{U}_t \approx -\frac{1}{2}\omega\left(\log P_t - \log\tilde{P}_t\right)^2 - \frac{1}{2}\eta(1-\eta+\eta\omega)\sigma^2_{\log p_{it}}$$

Rearranging and taking expectations:

$$\begin{aligned} \mathbb{E}_{t-1}\left\{\frac{U_t - \tilde{U}_t}{\eta(1-\eta+\eta\omega)}\right\} \approx& -\frac{1}{2}\left\{\left(\frac{\omega/\eta}{1-\eta+\eta\omega}\right)\mathbb{E}_{t-1}\left(\log P_t - \log\tilde{P}_t\right)^2 + \mathbb{E}_{t-1}\,\sigma^2_{\log p_{it}}\right\} \\ =& -\frac{1}{2}\left\{(1-\alpha^\star)\,\mathbb{E}_{t-1}\left(\log P_t - \log\tilde{P}_t\right)^2 + \mathbb{E}_{t-1}\,\sigma^2_{\log p_{it}}\right\} \end{aligned}$$

where $\alpha^\star := \frac{1+\eta\omega-\eta-\omega/\eta}{1+\eta\omega-\eta}$ is the efficient level of coordination. If $\kappa_1^{\mathcal{M}} \neq 0$, the economy is inefficient and the first-order term does not cancel out. Under the standard "small distortion" assumption, that is when the distortion has the same order of magnitude as fluctuations in volatility and dispersion, $\frac{1}{\mathcal{M}_t}$ can be ignored in the product with second-order terms. Let the efficient price level under complete information be $P_t^\star = \omega^{\frac{1}{\omega}}\frac{M}{A_t}$ and the efficient consumption $C_t^\star = \omega^{-\frac{1}{\omega}}A_t$. Ignoring terms of order higher than two:

$$\left(1 - \frac{1}{\mathcal{M}_t}\right)(-\hat{P}_t) \approx -\frac{1}{\mathcal{M}_t}\log(\mathcal{M}_t)\hat{P}_t \approx +\omega(\log P_t^\star - \log\tilde{P}_t)\hat{P}_t$$

This implies that $\left(1 - \frac{1}{\mathcal{M}_t}\right)(-\hat{P}_t)$ is increasing in ω, hence a large omega makes efficiency gains larger. It follows from equation (7) and the above that $\kappa_0 + \kappa_1\theta_t = -\log \tilde{P}_t = -(\log M + \frac{1}{\omega}\log(\omega\mathcal{M}_t) - \log A_t)$, and $\kappa_1 \equiv \frac{1}{\omega}\kappa_1^{\mathcal{M}} + \kappa_1^A$, whereas $\kappa_1^\star \equiv \kappa_1^A$. Using equation (16), the covariance term is positive whenever markups are countercyclical ($\kappa_1^{\mathcal{M}} > 0$). Putting everything together, and ignoring multiplicative constants, U_t can be approximated by

$$U_t - \tilde{U}_t \approx -\frac{1}{2}\left\{(1-\alpha^\star)\left[\mathbb{E}_{t-1}\left(\log\frac{P_t}{\tilde{P}_t}\right)^2 - 2\mathrm{Cov}_{t-1}\left(\log\frac{P_t^\star}{\tilde{P}_t}, \log\frac{P_t}{\tilde{P}_t}\right)\right] + \mathbb{E}_{t-1}\,\sigma^2_{\log p_{it}}\right\}$$

Moreover, ignoring terms that are independent of the communication policy, $\mathcal{U}_t \approx U_t - \tilde{U}_t$, because firms and household solve static problems and uncertainty is revealed at the end of each period. This representation shows that, up to a second-order approximation, the model is a special case of the abstract setting as in equation (6) with $k_i = -\log p_{it}$, $K = -\log P_t$, $\kappa = -\log \tilde{P}_t$, $\sigma_k^2 = \sigma^2_{\log p_{it}}$, $\kappa^\star = -\log P_t^\star$. Using the cash-in-hand constraint to substitute for the output gap leads to the formula in the text.

Comparative Statics on $\alpha^\star - \alpha$. Using the restriction on $\omega \geq \frac{2\eta-2}{2\eta-1}$ and taking derivatives

$$\begin{aligned}
\frac{\partial}{\partial\eta}\alpha^\star \geq 0 &\iff \omega \geq \frac{2\eta-1}{2\eta} \\
\frac{\partial}{\partial\eta}\alpha \geq 0 &\iff \omega \geq 1 \\
\frac{\partial}{\partial\eta}(\alpha^\star - \alpha) \geq 0 &\iff \eta \leq 2 \\
\frac{\partial}{\partial\omega}\alpha^\star &> 0 \\
\frac{\partial}{\partial\omega}\alpha &> 0 \\
\frac{\partial}{\partial\omega}(\alpha^\star - \alpha) &< 0 \\
\frac{\partial^2}{\partial\eta\partial\omega}(\alpha^\star - \alpha) \geq 0 &\iff \begin{cases}\omega \leq \frac{2\eta-\eta^2-1}{3\eta-\eta^2} & \eta < 3 \\ \forall\omega & \eta > 3\end{cases}
\end{aligned}$$

B.3 Proofs for Section 3

Proof of proposition 1. The result follows from theorem (1).

Proof of proposition 2. The result follows directly from lemma (3).

Proof of proposition 3. The result follows from theorem (2).

Proof of proposition 4. Let $\bar{\mu}_0$ and $1-\bar{\mu}_0$ be the zeros of the second derivative of welfare (by proof of theorem (3) we know that there are exactly two under condition (Inflation targeting)). If the optimal communication is partially revealing, for $\mu_0 > 1/2$ it must be

$$\mu_y = \begin{cases} 1-\bar{\mu}_0 & \text{if } \ y=1 \\ \mu_y^- & \text{if } \ y=0 \end{cases} \quad \tau = \begin{cases} \frac{\mu_0-\mu_y^-}{1-\bar{\mu}_0-\mu_y^-} & \text{if } \ \mu_y = 1-\bar{\mu}_0 \\ \frac{1-\bar{\mu}_0-\mu_0}{1-\bar{\mu}_0-\mu_y^-} & \text{if } \ \mu_y = \mu_y^- \end{cases}$$

The signal $\pi^y(y|s)$ that generated the posterior can be recovered from

$$\pi^y(y|\theta) = \sum_{s\in S} \pi^y(y|s)\pi^s(s|\theta) = \frac{\tilde{\mu}_y(\theta)\tau(\tilde{\mu}_y)}{\tilde{\mu}_0(\theta)}$$

Since $\mu_0 = \mu_y^-$, this implies that receiving the signal $y=0$ is fully informative, i.e. the planner never sends the signal $y=0$ upon observing $s=1$, hence $\pi^y(y=1|s=1)=1$. Rearranging the equation for the other signal

$$\pi^y(y=0|s=0) = \frac{\pi^y(y=0|\theta=0)}{q}$$

with $\pi^y(y=0|\theta=0) = \frac{(1-\mu_y^-)(1-\bar{\mu}_0-\mu_0)}{(1-\bar{\mu}_0-\mu_y^-)(1-\mu_0)} < q$. This implies $\pi^y(y=1|s=0) > 0$ and decreasing in $(1-\bar{\mu}_0-\mu_0)$, hence increasing in $(\mu_0-\frac{1}{2})$.

Proof of proposition 5 Using theorem (3) and corollary (5), condition (Decreasing at 0) is never satisfied for $\kappa_1^{\mathcal{M}} > 0$. Condition (Negative at 1/2) simplifies to

$$\frac{1}{(2p-1)^2} \leq \frac{(2\omega-1)(\eta-1)}{1+\eta\omega-\eta}$$

The LHS is decreasing in p and diverges to $+\infty$ for $p \to 1/2$ and is equal to 1 for $p=1$, hence the condition is violated for p small and is violated for all ps when $\eta < 2$. Condition (Convexity at $\{0,1\}$) simplifies to

$$-p(1-p) \geq \eta(2p-1)^2\left(\frac{\omega}{1+\eta\omega-\eta}\right)$$

which is never satisfied. Condition (Convexity at 1/2) simplifies to

$$\begin{aligned} &\omega \leq (\eta-1)[2\omega(2p-1)^2 - 4(\omega-1)p(1-p)] \\ \iff &\omega \leq \frac{(\eta-1)(4p(1-p))}{1+(\eta-1)(-12p^2+12p-2)} \end{aligned}$$

which is violated for $\eta \to 1$. Also the RHS is the second line is decreasing in p for $\eta < 3/2$ and increasing in p for $\eta > 3/2$. Hence the condition is violated for ω large, $\eta > 3/2$ and p small, or ω large, $\eta < 3/2$ and p large.

C Proofs for Main Text and Appendix A

C.1 Notation for the proofs

To simplify notation in the proofs, let $\mu \equiv \mu_y$ be the posterior probability that the state is high, conditional on observing the public signal realization y, $\mathbb{E}_\mu \equiv \mathbb{E}_{\theta \sim \tilde{\mu}_y}$ the posterior expectation, and $\pi_\mu(x)$ the posterior probability of observing signal x when the posterior probability on θ is $\tilde{\mu}(\theta) \equiv \tilde{\mu}_y(\theta)$. Conditional probabilities on the private signal will be denoted accordingly. From Bayes rule it is immediate to show that the posterior expectation given the private and public signal is:

$$\mathbb{E}_\mu\{\theta|x\} = \begin{cases} \frac{p\mu}{p\mu+(1-p)(1-\mu)} & x=1 \\ \\ \frac{(1-p)\mu}{(1-p)\mu+p(1-\mu)} & x=0 \end{cases}$$

And the conditional probability that another agent observed realization x' of the private signal conditional on observing x is:

$$\pi_\mu(x'=1|x) = \sum_\theta \pi^x(x'=1|\theta)\tilde{\mu}(\theta|x) =: \begin{cases} \bar{\pi}_\mu = \frac{p^2\mu+(1-p)^2(1-\mu)}{p\mu+(1-p)(1-\mu)} & x=1 \\ \\ \underline{\pi}_\mu = \frac{p(1-p)}{(1-p)\mu+p(1-\mu)} & x=0 \end{cases}$$

C.2 Proof of proposition 6

Consider the symmetric strategy:

$$k(x,\mu) = \begin{cases} \kappa_0 + \kappa_1 \bar{k}(\mu) & x=1 \\ \\ \kappa_0 + \kappa_1 \underline{k}(\mu) & x=0 \end{cases}$$

Then the expectation of the average action is:

$$\mathbb{E}_\mu\left\{K(\theta,\mu)|x\right\} = \mathbb{E}_\mu\left\{\left(\int_0^1 k_i di\right)|x\right\} = \int_0^1 \mathbb{E}_\mu\{k(x_i,\mu)|x\}di = \mathbb{E}_\mu\{k(x',\mu)|x\}$$

Hence:

$$\mathbb{E}_\mu\left\{K(\theta,\mu)|x\right\} = \begin{cases} \kappa_0 + \kappa_1\big(\bar{\pi}_\mu \bar{k}(\mu) + (1-\bar{\pi}_\mu)\underline{k}(\mu)\big) & x=1 \\ \\ \kappa_0 + \kappa_1\big(\underline{\pi}_\mu \bar{k}(\mu) + (1-\underline{\pi}_\mu)\underline{k}(\mu)\big) & x=0 \end{cases}$$

For this strategy to be an equilibrium, it must hold that the guessed solution is a best response:

$$k(x,\mu) = (1-\alpha)\left[\kappa_0 + \kappa_1 \mathbb{E}_\mu\{\theta|x\}\right] + \alpha\,\mathbb{E}_\mu\left\{K|x\right\}$$

$$\Rightarrow \bar{k}(\mu) = (1-\alpha)\,\mathbb{E}_\mu\{\theta|x=1\} + \alpha\,\mathbb{E}_\mu\{K(\theta,\mu)|x=1\}$$
$$\underline{k}(\mu) = (1-\alpha)\,\mathbb{E}_\mu\{\theta|x=0\} + \alpha\,\mathbb{E}_\mu\{K(\theta,\mu)|x=0\}$$

This is a linear system of two equations in two unknowns, that can be solved analytically for $\bar{k}(\mu)$ and $\underline{k}(\mu)$. The solutions are:

$$\bar{k}(\mu) = \frac{-(1-\alpha)p(2p-1)\mu^2 + p(\alpha(1-p)+(1-\alpha)p)\mu}{(1-\alpha)(2p-1)^2\mu(1-\mu)+p(1-p)} \tag{8}$$

$$\underline{k}(\mu) = \frac{(1-\alpha)(1-p)(2p-1)\mu^2 + (1-p)(\alpha p+(1-\alpha)(1-p))\mu}{(1-\alpha)(2p-1)^2\mu(1-\mu)+p(1-p)} \tag{9}$$

Collecting terms leads to

$$\bar{k}(\mu) \equiv \frac{1 + \left(\frac{\alpha}{1-\alpha}\right)\frac{\mathbb{E}_\mu\{\theta|x=0\}}{\mu}}{\frac{1}{\mathbb{E}_\mu\{\theta|x=1\}} + \left(\frac{\alpha}{1-\alpha}\right)\frac{\mathbb{E}_\mu\{\theta|x=0\}}{\mu^2}}$$

Adding and subtracting $\kappa_1\,\mathbb{E}_\mu\{\theta|x=1\}$ results in the formula. The same steps lead to $\underline{k}(\mu)$ as well. Uniqueness follows from the argument in Morris and Shin (2002) and the restrictions on the parameter $\alpha \in (-1,1)$.

C.3 Proof of lemma 1

Recall that in equation (2), it was possible to rewrite the equilibrium strategy as a function of the posterior induced by the signal. This implies that also welfare is a function of the posterior belief. In particular, it simplifies to:

$$\mathbb{E}\,\hat{u}(\pi^y) = \mathbb{E}\,u(\mu_y) := \mathbb{E}_{\theta\sim\tilde{\mu}_y}\,\mathbb{E}_{x\sim\pi^x(\theta)}\,U\big(k(x,\mu_y),K(\theta,\mu_y),\sigma_k(\theta,\mu_y),\theta\big)$$

where $k(x,\mu_y)$ is the equilibrium strategy, $K(\theta,\mu_y) \equiv \sum_x k(x,\mu_y)\pi^x(x|\theta)$, and $\sigma_k(\theta,\mu_y) \equiv \big(\sum_x (k(x,\mu_y)-K(\theta,\mu_y))^2\pi^x(x|\theta)\big)^{1/2}$ for all (θ,μ_y).

This simplifies drastically the problem, because maximization can now be performed over the possible posteriors induced by the public signal. The idea is that, when the planner chooses some signal π^y, each signal realization y leads to a posterior belief $\tilde{\mu}_y$. Therefore, from an ex ante perspective, one can think of the choice of π^y as inducing a distribution of posteriors.

Let τ be the distribution of posteriors induced by the signal π^y. τ is consistent with bayesian updating, i.e. it is Bayes plausible, if the average over signal realizations is equal to the prior. The following definition clarifies the terms introduced in the binary-state context.

Definition 4 (Bayes-plausible distribution of posteriors)**.** *Let*

$$\tau(\mu_y) \equiv \sum_{y\in Y}\sum_{\theta\in\Theta}\pi^y(y|\theta)\tilde{\mu}_0(\theta)$$

be the distribution of posteriors induced by the signal π^y. τ is Bayes plausible if

$$\mathbb{E}_{\mu_y\sim\tau}\,\mu_y = \mu_0 \tag{10}$$

Given a posterior belief and a Bayes-plausible distribution, it is immediate to recover the signal that generated the distribution from:

$$\pi^y(y|\theta) = \frac{\tilde{\mu}_y(\theta)\tau(\mu_y)}{\tilde{\mu}_0(\theta)} \tag{11}$$

Combining all the above leads to the following reformulation of planner's problem, for the case in which $q = 1$:

Lemma 5 (Planner's problem)**.** *When $q = 1$, the planner's problem in (Planner's problem) is equivalent to:*

$$\begin{aligned} &\max_{\tau} \mathbb{E}_{\mu_y \sim \tau} \, \mathbb{E}\, u(\mu_y) \\ &\textit{subject to} \quad \mathbb{E}_{\mu_y \sim \tau} \, \mu_y = \mu_0 \end{aligned} \tag{12}$$

Proof. The proof is standard and follows immediately from Kamenica and Gentzkow (2011). □

Let us now move to the case in which $q \leq 1$. From Bayes rule, the posterior probability of the state after observing the public signal is given by:

$$\tilde{\mu}(\theta) = \frac{\sum_s \pi^y(y|s)\pi^s(s|\theta)\tilde{\mu}_0(\theta)}{\sum_{\theta'}\sum_{s'} \pi^y(y|s')\pi^s(s'|\theta')\tilde{\mu}_0(\theta')} \tag{13}$$

The upper bound can be found by solving the problem:

$$\max_{\{\pi^y(y|s)\}_{s=0,1}} \tilde{\mu}(\theta = 1)$$

Since $q > 1/2$, the solution is given by $\pi^y(y = 1|s = 1) = \pi^y(y = 0|s = 0) = 1$, which is also the solution of the equivalent minimization problem for the lower bound. Plugging the marginal distributions into equation (13) gives the bounds. The rest of the proof follows directly from Lemma (5).

C.4 Proof of lemma 2

Follows from lemma (4) with $\kappa = \kappa^\star$.

C.5 Proof of theorem 1

Let us start by proving the theorem for $q = 1$.

Proposition 8. *In economies with $\kappa = \kappa^\star$ and $q = 1$, the optimal communication policy is fully revealing. That is*

$$\mu_y = \begin{cases} 1 & \textit{if } \ y = 1 \\ 0 & \textit{if } \ y = 0 \end{cases} \quad \tau = \begin{cases} \mu_0 & \textit{if } \ \mu_y = 1 \\ 1 - \mu_0 & \textit{if } \ \mu_y = 0 \end{cases}$$

and $\pi^y(y = 1|\theta = 1) = \pi^y(y = 0|\theta = 0) = 1$.

Proof. Since $W_{KK} < 0$ and $W_{\sigma\sigma} < 0$, it follows from equation (14) that $\mathbb{E}\, u \leq \mathbb{E}\, W(\kappa^\star, 0, \theta)$ and the maximum is achieved at $k(x, \mu) = \kappa^\star(\theta)$ for all (x, μ). For the proof to be complete it is enough to notice that

$$k(x, \mu) = \begin{cases} \kappa_0 + \kappa_1 & \mu = 1 \\ \kappa_0 & \mu = 0 \end{cases}$$

This can be verified directly by plugging μ into equations (8)-(9). It follows that $k(x, \mu) = \kappa(\theta)$ for all (x, μ) provided that $\mu = 1$ if and only if $\theta = 1$. Since the economy is efficient, $k(x, \mu) = \kappa^\star(\theta)$ for all (x, μ) under the same condition. Finally, $\mu = 1$ if and only if $\theta = 1$ can be implemented with a fully informative signal, and the Bayes-plausibility constraint is satisfied by setting $\tau(\mu_H) = \mu_0$. □

The rest of the proof for $q < 1$ is a direct consequence of the general characterization in theorem (3). In particular, it is enough to prove that welfare is twice-continuously differentiable in μ, which follows from lemma (8), the fact that the derivative of welfare around $\mu \in \{0, 1\}$ is strictly negative (in the proof of lemma (9)), and that welfare is always strictly below zero (ensured by condition (Negative at 1/2)). This leads to the solution of the simplified problem, that is

$$\mu_y = \begin{cases} \mu_y^+ & \text{if } \; y = 1 \\ \mu_y^- & \text{if } \; y = 0 \end{cases} \quad \tau = \begin{cases} \frac{\mu_0 - \mu_y^-}{\mu_y^+ - \mu_y^-} & \text{if } \; \mu_y = \mu_y^+ \\ \frac{\mu_y^+ - \mu_0}{\mu_y^+ - \mu_y^-} & \text{if } \; \mu_y = \mu_y^- \end{cases}$$

This implies that the optimal policy is fully revealing.

C.6 Proof of theorem 2

The proof follows from the characterization of curvature of welfare derived generally in theorem (3) and the intuition from figure (1). Conditions (Concavity at $\{0, 1\}$)-(Concavity at 1/2) are necessary and sufficient for welfare to be concave in (possibly degenerate) intervals $[0, \mu^\ell] \cup [\mu^h, 1]$ and $[\mu^\ell, \mu^h]$ respectively, where μ^ℓ and μ^h are the zeros of the second derivative. Moreover, given symmetry of welfare, also those intervals are symmetric around 1/2, hence $\mu^\ell = 1 - \mu^h$.

Let us now prove the first bullet point of the proposition. Under condition (Concavity at $\{0, 1\}$), welfare is strictly concave over $[0, \mu^\ell] \cup [\mu^h, 1]$ from condition (Convexity at $\{0, 1\}$) evaluated at $\kappa_1 = \kappa_1^\star$. Hence there exists a q small enough such that welfare is concave in $[\mu^-, \mu^+]$ (unless $\mu_0 \in \{\mu^\ell, \mu^h\}$, but then there exists a $\underline{q}$ that gives indifference). Hence the policy is non-disclosing. For the only if part, when the policy is revealing and q is sufficiently small, it must be the case that welfare is convex, otherwise a non-disclosing policy would be optimal by Jensen's inequality. The second bullet point uses the same argument, replacing condition (Convexity at $\{0, 1\}$) with (Convexity at 1/2).

For the third bullet point, notice than in those cases that do not follow neither under the first bullet point nor under theorem (1), that is for intermediate

qs, $[\mu^-, \mu^+]$ can be large enough such that welfare is concave-convex or convex-concave over $[\mu^-, \mu^+]$. If the policy is partially revealing and $\mu_0 \in [\mu^\ell, \mu^h]$, it must be the case that welfare is concave-convex-concave and $[\mu^\ell, \mu^h]$ is the subset of the domain on which welfare is convex. This follows from conditions (Decreasing at 0)-(Negative at 1/2) and the fact that welfare has at most two zeros ($\{\mu^\ell, \mu^h\}$) in the second derivative. This implies condition (Concavity at $\{0,1\}$) on the concave part. For the forth bullet point, the argument is the same with conditions (Convexity at $\{0,1\}$)-(Convexity at 1/2) instead.

C.7 Proof of lemma 3

Following the steps in the proof of theorem (3), after much algebra, the second derivatives at $\mu \in \{0, .5, 1\}$. The second derivative of dispersion around $\mu \in \{0,1\}$ is

$$\frac{\partial^2\,\mathbb{E}[(k-K)^2]}{\partial\mu^2} = 2\kappa_1^2 \frac{(1-\alpha)^2(2p^2-3p+1)^2}{p(1-p)^3} > 0$$

The second derivative of volatility is

$$\frac{\partial^2\,\mathbb{E}[(K-\kappa)^2]}{\partial\mu^2} = 2\kappa_1^2 \frac{(1-\alpha)(7p(1-p)-2) - \alpha p(1-p)}{p(1-p)}$$

which is always negative for the parameter values allowed because

$$\frac{7p(1-p)-2}{p(1-p)} < -1 < -\frac{1}{2} < \frac{\alpha}{1-\alpha}$$

Therefore, changing sign (from $W_{\sigma\sigma} < 0$), it follows from lemma (8) that the negative of dispersion is always concave in a neighborhood of $\{0,1\}$, and the negative of volatility is always convex. For $\mu = \frac{1}{2}$, similar calculations lead to

$$\text{sign}\left(\frac{\partial^2\,\mathbb{E}[(k-K)^2]}{\partial\mu^2}\right) = \text{sign}\left(-\alpha p(1-p) - \frac{(1-\alpha)}{4}\right) = -1$$

because $-\alpha p(1-p) - \frac{(1-\alpha)}{4} < p(1-p) - \frac{1}{2} < 0$. Hence the negative of dispersion is always convex around .5. For volatility, the sign is ambiguous:

$$\text{sign}\left(\frac{\partial^2\,\mathbb{E}[(K-\kappa)^2]}{\partial\mu^2}\right) = \text{sign}\left(p^2(1-p)^2(\alpha(\alpha-2)) + \frac{1}{2}\left(\frac{1}{8} - p(1-p)\right)(\alpha(\alpha-2)+1)\right)$$

that is the second derivative is positive if and only if

$$\frac{\alpha(\alpha-2)}{\alpha(\alpha-2)+1} > \frac{1}{2}\frac{p(1-p)-\frac{1}{8}}{p^2(1-p)^2}$$

Hence, the negative of volatility is concave if and only if the condition holds. The LHS is decreasing in α, and the RHS is decreasing in p and going to $-\infty$ for $p \to 1$. Hence there exist an α small enough and p large enough such that the negative of volatility is concave. Finally, using the fact that dispersion and volatility are fourth-order polynomials in μ with at most two zeros in the second derivative, and they are symmetric around .5, this proves the lemma.

C.8 Proof of lemma 4

It follows immediately from proposition 8 in Angeletos and Pavan (2007) that utility can be rewritten as (notice that $\mathbb{E}(K-\kappa)=0$):

$$\begin{aligned}\mathbb{E}\,u =&\,\mathbb{E}\,W(\kappa,0,\theta) - W_{KK}\mathrm{Cov}(K-\kappa,\kappa^\star-\kappa)\\ &+\frac{W_{KK}}{2}\,\mathbb{E}[(K-\kappa)^2]+\frac{W_{\sigma\sigma}}{2}\,\mathbb{E}[(k-K)^2]\end{aligned} \tag{14}$$

where the expectation $\mathbb{E}$ is taken with respect to both $\tilde{\mu}$ and $\pi^x(\theta)$. See appendix A for the definition of utilitarian welfare W. Also, $W_{KK}\equiv U_{kk}+2U_{kK}+U_{KK}<0$ and definition of $\alpha^\star$ allow to replace W_{KK} for $W_{\sigma\sigma}$. Lastly, notice that $\mathbb{E}\,W(\kappa,0,\theta)$ is constant (across all Bayes-plausible τs) because

$$\mathbb{E}\,W(\kappa,0,\theta)=\mathbb{E}\,W(0,0,0)+W_K\,\mathbb{E}\,\kappa+W_\theta\,\mathbb{E}\,\theta+\frac{1}{2}W_{KK}\,\mathbb{E}\,\kappa^2+W_{\theta\theta}\,\mathbb{E}\,\theta^2$$

where κ is linear and $\mathbb{E}\,\theta^2=\mu(1-\mu)+\mu^2=\mu$.

C.9 Proof of theorem 3

The proof will proceed in several steps. First, rewrite equation (14) as a quadratic function of $\bar{k}$ and $\underline{k}$. Second, derive first-order conditions to simplify derivatives of welfare with respect to the parameter μ (envelope theorem). Third, check that welfare is twice continuously differentiable and take derivatives, using the envelope theorem to make simplifications. Fourth, evaluate the second derivative at $\mu\in\{0,\frac{1}{2},1\}$ to study locally its concavity/convexity. Given the fact that welfare is twice continuously differentiable, symmetric, and with (at most) two zeros in the second derivative, concavity/convexity of welfare at those three points characterizes fully concavity/convexity of welfare. Finally, for the global characterization of the policy, one needs to check when the concavification coincides with the smallest concave function that dominates welfare.

Lemma 6. *$\mathbb{E}\,u-\bar{u}$ is a quadratic function of $(\bar{k},\underline{k})$ proportional to:*

$$\mathbb{E}\,u-\bar{u}\propto-(\bar{k},\underline{k})'Q^\star(\bar{k},\underline{k})+2\left(\frac{\kappa_1^\star}{\kappa_1}\right)L(\bar{k},\underline{k})$$

where $Q^\star(\mu)$ is a 2×2 matrix and $L(\mu)$ is a 1×2 matrix with entries:

$$\begin{aligned}q_{11}^\star&\equiv\mu p^2+(1-\mu)(1-p)^2+\frac{p(1-p)}{1-\alpha^\star}\\ q_{12}^\star=q_{21}^\star&\equiv-\left(\frac{\alpha^\star}{1-\alpha^\star}\right)p(1-p)\\ q_{22}^\star&\equiv\mu(1-p)^2+(1-\mu)p^2+\frac{p(1-p)}{1-\alpha^\star}\\ \ell_1&\equiv\mu p\\ \ell_2&\equiv\mu(1-p)\end{aligned}$$

Proof. Replace W_{KK} using the definition of $\alpha^\star$. The second derivative of $\mathbb{E}\,u$ has the same sign as the second derivative of

$$(1-\alpha^\star)\left(2\frac{\mathrm{Cov}(K-\kappa,\kappa^\star-\kappa)}{\kappa_1^2}-\mathbb{E}\left[\left(\frac{K-\kappa}{\kappa_1}\right)^2\right]\right)-\mathbb{E}\left[\left(\frac{k-K}{\kappa_1}\right)^2\right] \tag{15}$$

Now, let's analyze term by term. Since

$$\frac{k-K}{\kappa_1}=\begin{cases}\bar{k}-p\bar{k}-(1-p)\underline{k}=(1-p)(\bar{k}-\underline{k}) & x=1,\theta=1\\ \underline{k}-p\bar{k}-(1-p)\underline{k}=-p(\bar{k}-\underline{k}) & x=0,\theta=1\\ \bar{k}-(1-p)\bar{k}-p\underline{k}=p(\bar{k}-\underline{k}) & x=1,\theta=0\\ \underline{k}-(1-p)\bar{k}-p\underline{k}=-(1-p)(\bar{k}-\underline{k}) & x=0,\theta=0\end{cases}$$

this implies that $\sigma_k^2=p(1-p)\kappa_1^2(\bar{k}-\underline{k})^2$, which is constant across θs. Hence the dispersion term is

$$\mathbb{E}\left[\left(\frac{k-K}{\kappa_1}\right)^2\right]=\mathbb{E}\left[\frac{\sigma_k^2}{\kappa_1^2}\right]=p(1-p)(\bar{k}-\underline{k})^2$$

Also,

$$\frac{K-\kappa}{\kappa_1}=\begin{cases}p\bar{k}+(1-p)\underline{k}-1 & \theta=1\\ (1-p)\bar{k}+p\underline{k} & \theta=0\end{cases}$$

This leads to the formula for the non-fundamental volatility

$$\begin{aligned}\mathbb{E}\left[\left(\frac{K-\kappa}{\kappa_1}\right)^2\right]&=2p(1-p)\bar{k}\underline{k}+\bar{k}^2\{\mu p^2+(1-\mu)(1-p)^2\}\\&\quad+\underline{k}^2\{\mu(1-p)^2+(1-\mu)p^2\}+\mu-2\mu\{p\bar{k}+(1-p)\underline{k}\}\end{aligned}$$

Finally, the covariance term is

$$\mathbb{E}\left[\frac{(K-\kappa)\cdot(\kappa^\star-\kappa)}{\kappa_1^2}\right]=-\mu\left(\frac{\kappa_1^\star-\kappa_1}{\kappa_1}\right)[1-p\bar{k}-(1-p)\underline{k}] \tag{16}$$

Since $1-p\bar{k}-(1-p)\underline{k}>0$, the covariance is positive if and only if $\kappa_1^\star<\kappa_1$.

To get the quadratic form, collect terms and rearrange. This leads to the planner's objective function to be minimized:

$$\begin{aligned}&(1-\alpha^\star)\left\{\mathbb{E}\left[\left(\frac{K-\kappa}{\kappa_1}\right)^2\right]-2\,\mathbb{E}\left[\frac{(K-\kappa)\cdot(\kappa^\star-\kappa)}{\kappa_1^2}\right]\right\}+\mathbb{E}\left[\left(\frac{k-K}{\kappa_1}\right)^2\right]\\&=(1-\alpha^\star)\Big\{\bar{u}+(\bar{k},\underline{k})'Q^\star(\bar{k},\underline{k})-2\left(\frac{\kappa_1^\star}{\kappa_1}\right)L(\bar{k},\underline{k})\Big\}\end{aligned} \tag{17}$$

where $\bar{u}\equiv 2\mu\frac{\kappa_1^\star}{\kappa_1}-\mu$ and $Q^\star$, L are as in the proposition. $\square$

Let us start to derive the envelope conditions for planner's problem. Since k is possibly different from $k^\star$, this only applies to the case in which $\alpha = \alpha^\star$ and $\kappa = \kappa^\star$.

Lemma 7. *The first order conditions for the decentralized economy are:*

$$\begin{aligned} q_{11}\bar{k} + q_{12}\underline{k} - \ell_1 = 0 \\ q_{22}\underline{k} + q_{12}\bar{k} - \ell_2 = 0 \end{aligned} \tag{18}$$

Proof. When $\alpha = \alpha^\star$ and $\kappa = \kappa^\star$, planner's objective function simplifies to:

$$(1-\alpha)\,\mathbb{E}\left[\left(\frac{K-\kappa}{\kappa_1}\right)^2\right] + \mathbb{E}\left[\left(\frac{k-K}{\kappa_1}\right)^2\right] = (1-\alpha)\Big\{\bar{u} + (\bar{k},\underline{k})'Q(\bar{k},\underline{k}) - 2L(\bar{k},\underline{k})\Big\}$$

where

$$\begin{aligned} q_{11} &\equiv \mu p^2 + (1-\mu)(1-p)^2 + \frac{p(1-p)}{1-\alpha} \\ q_{12} = q_{21} &\equiv -\left(\frac{\alpha}{1-\alpha}\right)p(1-p) \\ q_{22} &\equiv \mu(1-p)^2 + (1-\mu)p^2 + \frac{p(1-p)}{1-\alpha} \\ \ell_1 &\equiv \mu p \\ \ell_2 &\equiv \mu(1-p) \\ \bar{u} &\equiv \mu \end{aligned}$$

Taking derivatives gives the FOCs in equation (18).[45] □

Let us now check that welfare is twice continuously differentiable in μ over $(0,1)$. Given lemma (6), it is enough to check that $\bar{k}(\mu)$ and $\underline{k}(\mu)$ –or equivalently $k(x,\mu)$– are twice continuously differentiable in μ over $(0,1)$.

Lemma 8. $k(x,\mu)$ *is twice continuously differentiable in* μ *over* $(0,1)$*.*

Proof. Using equations (8) and (9), after some tedious algebra, one can show that the first derivatives of $\bar{k}$ and $\underline{k}$ with respect to μ are:

$$\bar{k}' = \frac{p(1-p)\Big[(2p-1)\Big(2\mu^2 p - (1-\alpha)(2\mu p + \mu^2) - \alpha p(1+2\mu^2)\Big) + p^2\Big]}{\Big((1-\alpha)(2p-1)^2\mu(1-\mu) + p(1-p)\Big)^2} \tag{19}$$

$$\underline{k}' = \frac{p(1-p)\Big[(1-\alpha)\Big(\mu(1-\mu)(4p(1-p)-1) + 2\mu p + 1 - \mu\Big) + \alpha(3p-2p^2) - p(2-p)\Big]}{\Big((1-\alpha)(2p-1)^2\mu(1-\mu) + p(1-p)\Big)^2} \tag{20}$$

[45] As a check, one can solve this system and show that it leads to equations (8)-(9).

The second derivatives are:

$$\bar{k}'' = \frac{2p(2\mu-1)(2p-1)^2(1-\alpha)(1-p)\Big[(2p-1)\Big(2\mu^2p-\mu(\mu+2p)(1-\alpha)-\alpha p(2\mu^2+1)\Big)+p^2\Big]}{\Big((1-\alpha)(2p-1)^2\mu(1-\mu)+p(1-p)\Big)^3} - \frac{2p(2p-1)(1-\alpha)(1-p)(\mu+p-2\mu p)}{\Big((1-\alpha)(2p-1)^2\mu(1-\mu)+p(1-p)\Big)^2} \tag{21}$$

$$\underline{k}'' = \frac{2p(2\mu-1)(2p-1)^2(1-\alpha)(1-p)\Big[(1-\alpha)\big(\mu(1-\mu)(4p(1-p)-1)+\mu(2p-1)+1\big)\Big]}{\Big((1-\alpha)(2p-1)^2\mu(1-\mu)+p(1-p)\Big)^3} + \frac{2p(2\mu-1)(2p-1)^2(1-\alpha)(1-p)\Big[p(p-2)-\alpha p(2p-3)\Big]}{\Big((1-\alpha)(2p-1)^2\mu(1-\mu)+p(1-p)\Big)^3} - \frac{2p(2p-1)(1-\alpha)(1-p)(\mu+p-2\mu p-1)}{\Big((1-\alpha)(2p-1)^2\mu(1-\mu)+p(1-p)\Big)^2} \tag{22}$$

For $\bar{k}$ and $\underline{k}$ to be twice continuously differentiable, it is enough that the denominator is non-zero:

$$(1-\alpha)(2p-1)^2\mu(1-\mu)+p(1-p)\neq 0$$

Roots of this equations are

$$\mu = \frac{(1-\alpha)(2p-1)\pm\sqrt{(1-\alpha)(1-4\alpha p^2+4\alpha p-\alpha)}}{2(1-\alpha)(2p-1)} \notin (0,1)$$

The larger root is ≤ 1 provided that $p(1-p)\leq 0$, the smaller root is ≥ 0 provided that $(1-\alpha)p^2+\alpha p\leq 0$. Hence for $p\in(0,1)$, k is twice continuously differentiable. □

Lemma 9. *There exist (possibly degenerate) intervals* $[0,\mu^\ell]$, $[\underline{\mu}^m,\bar{\mu}^m]$, *and* $[\mu^h,1]$ *with* $0\leq\mu^\ell\leq\underline{\mu}^m\leq\frac{1}{2}\leq\bar{\mu}^m\leq\mu^h\leq 1$, *such that:*

- *For* $\mu\in[0,\mu^\ell]\cup[\mu^h,1]$, *welfare is (weakly) convex if and only if*

$$\frac{2\kappa_1^\star-\kappa_1}{\kappa_1}\geq -\left(\frac{\alpha-\alpha^\star}{1-\alpha^\star}\right)\left(\frac{(2p-1)^2(1-\alpha)}{(2p-1)^2(1-\alpha)+p(1-p)}\right)$$

- *For* $\mu\in[\underline{\mu}^m,\bar{\mu}^m]$, *welfare is (weakly) convex if and only if*

$$\frac{2\kappa_1^\star-\kappa_1}{\kappa_1}\geq\left(\frac{\alpha-\alpha^\star}{1-\alpha^\star}\right)\left(\frac{2(2p-1)^2(1-\alpha)}{(1-\alpha)+4\alpha p(1-p)}\right)$$

Proof. The first step is to take derivatives of welfare with respect to μ, using the quadratic representation introduced. Let us now rewrite $-(\bar{k},\underline{k})'Q^\star(\bar{k},\underline{k}) + 2\left(\frac{\kappa_1^\star}{\kappa_1}\right)L(\bar{k},\underline{k})$ from lemma (4) as

$$
\begin{aligned}
& -(\bar{k},\underline{k})'Q(\bar{k},\underline{k}) + 2L(\bar{k},\underline{k}) \\
& -(\bar{k},\underline{k})'(Q^\star - Q)(\bar{k},\underline{k}) + 2L(\bar{k},\underline{k})\left(\frac{\kappa_1^\star - \kappa_1}{\kappa_1}\right)
\end{aligned}
$$

where

$$
Q^\star - Q = \underbrace{\frac{\alpha^\star - \alpha}{(1-\alpha^\star)(1-\alpha)}p(1-p)}_{\equiv \tilde{q}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}
$$

Notice also that the derivative with respect to μ of Q and $Q^\star$ implies that $q'_{11} = -q'_{22} = 2p-1$ and $q'_{12} = \tilde{q}' = 0$.

Hence, using the first-order conditions in equation (18) to simplify, the first derivative of $-(\bar{k},\underline{k})'Q^\star(\bar{k},\underline{k}) + 2\left(\frac{\kappa_1^\star}{\kappa_1}\right)L(\bar{k},\underline{k})$ with respect to μ is

$$
\begin{aligned}
\frac{\partial}{\partial\mu} = & -(2p-1)\big(\bar{k}^2 - \underline{k}^2\big) - 2\tilde{q}(\bar{k}-\underline{k})(\bar{k}'-\underline{k}') \\
& + 2\left(\frac{\kappa_1^\star}{\kappa_1}\right)(p\bar{k} + (1-p)\underline{k}) + 2\mu\left(\frac{\kappa_1^\star - \kappa_1}{\kappa_1}\right)(p\bar{k}' + (1-p)\underline{k}')
\end{aligned}
$$

The second derivative is:

$$
\begin{aligned}
\frac{\partial^2}{\partial\mu^2} = & -2(2p-1)\big(\bar{k}\bar{k}' - \underline{k}\underline{k}'\big) - 2\tilde{q}\big((\bar{k}'-\underline{k}')^2 + (\bar{k}-\underline{k})(\bar{k}''-\underline{k}'')\big) \\
& 2\mu\left(\frac{\kappa_1^\star - \kappa_1}{\kappa_1}\right)(p\bar{k}'' + (1-p)\underline{k}'') + 2\left(\frac{2\kappa_1^\star - \kappa_1}{\kappa_1}\right)(p\bar{k}' + (1-p)\underline{k}')
\end{aligned}
$$

The derivatives above coincides with that of welfare only up to a multiplicative positive constant $\kappa_1^2 p(1-p)$, and the first derivative is also missing the derivative of the $\bar{u}$ term $-\frac{2\kappa_1^\star - \kappa_1}{\kappa_1}$ (only the second derivative is zero). Re-scaling appropriately, the derivatives of welfare are:

$$
\begin{aligned}
\frac{\partial}{\partial\mu}\mathbb{E}\,u = & \kappa_1^2(1-\alpha^\star)\Big\{ -\frac{2\kappa_1^\star - \kappa_1}{\kappa_1} - (2p-1)\big(\bar{k}^2 - \underline{k}^2\big) - 2\tilde{q}(\bar{k}-\underline{k})(\bar{k}'-\underline{k}') \\
& + 2\left(\frac{\kappa_1^\star}{\kappa_1}\right)(p\bar{k} + (1-p)\underline{k}) + 2\mu\left(\frac{\kappa_1^\star - \kappa_1}{\kappa_1}\right)(p\bar{k}' + (1-p)\underline{k}')\Big\} \\
\frac{\partial^2}{\partial\mu^2}\mathbb{E}\,u = & \kappa_1^2(1-\alpha^\star)\Big\{ -2(2p-1)\big(\bar{k}\bar{k}' - \underline{k}\underline{k}'\big) - 2\tilde{q}\big((\bar{k}'-\underline{k}')^2 + (\bar{k}-\underline{k})(\bar{k}''-\underline{k}'')\big) \\
& 2\mu\left(\frac{\kappa_1^\star - \kappa_1}{\kappa_1}\right)(p\bar{k}'' + (1-p)\underline{k}'') + 2\left(\frac{2\kappa_1^\star - \kappa_1}{\kappa_1}\right)(p\bar{k}' + (1-p)\underline{k}')\Big\}
\end{aligned}
$$

The last step is to evaluate welfare and derivatives of welfare at $\mu \in \{0, \frac{1}{2}, 1\}$. Assuming $\mu = 1$ and plugging into the formulas derived before, one can obtain:

$$\begin{aligned}
\bar{k} =& 1 \\
\bar{k}' =& (1-\alpha)\frac{1-p}{p} + \alpha \\
\bar{k}'' =& \frac{2(2p-1)(1-\alpha)(\alpha + 2p - 4\alpha p + 4\alpha p^2 - p^2 - 1)}{p^2(1-p)} \\
\underline{k} =& 1 \\
\underline{k}' =& \frac{\alpha + p - 2\alpha p}{1-p} \\
\underline{k}'' =& -\frac{2(2p-1)(1-\alpha)(\alpha - 4\alpha p + 4\alpha p^2 - p^2)}{p(1-p)^2} \\
\mathbb{E}\, u =& 0 \\
\frac{\partial}{\partial \mu}\mathbb{E}\, u =& \kappa_1^2(1-\alpha^\star)\frac{2\kappa_1^\star - \kappa_1}{\kappa_1} \\
\frac{\partial^2}{\partial \mu^2}\mathbb{E}\, u =& 2\kappa_1^2(1-\alpha^\star)\left\{\frac{(2p-1)^2(1-\alpha)}{p(1-p)}\left(\frac{2\kappa_1^\star - \kappa_1}{\kappa_1} - \frac{\alpha^\star - \alpha}{1-\alpha^\star}\right) + \frac{2\kappa_1^\star - \kappa_1}{\kappa_1}\right\}
\end{aligned}$$

The first derivative is positive provided that $\kappa_1 > 0$ and $\kappa_1^\star > \frac{1}{2}\kappa_1$ or $\kappa_1 < 0$ and $\kappa_1^\star < \frac{1}{2}\kappa_1$. The second derivative is positive whenever

$$\frac{2\kappa_1^\star - \kappa_1}{\kappa_1} \geq \left(\frac{\alpha^\star - \alpha}{1-\alpha^\star}\right)\left(\frac{(2p-1)^2(1-\alpha)}{(2p-1)^2(1-\alpha) + p(1-p)}\right)$$

The first term on the RHS vanishes for $\alpha^\star \to \alpha$, the second term vanishes for either $p \to \frac{1}{2}$ or $\alpha \to 1$. The LHS vanishes for $\kappa_1^\star \to \frac{1}{2}\kappa_1$. Since k is continuous with continuous derivatives, $\mathbb{E}\, u$ is continuous with continuous derivatives, which proves the result in a neighborhood of $\mu = 1$.

Let us now move to the case in which $\mu = 0$:

$$\begin{aligned}
\bar{k} =& 0 \\
\bar{k}' =& \frac{\alpha + p - 2\alpha p}{1-p} \\
\bar{k}'' =& -\frac{2(2p-1)(1-\alpha)(\alpha - 4\alpha p + 4\alpha p^2 - p^2)}{p(1-p)^2} \\
\underline{k} =& 0 \\
\underline{k}' =& (1-\alpha)\frac{1-p}{p} + \alpha \\
\underline{k}'' =& \frac{2(2p-1)(1-\alpha)(\alpha + 2p - 4\alpha p + 4\alpha p^2 - p^2 - 1)}{p^2(1-p)} \\
\mathbb{E}\, u =& 0 \\
\frac{\partial}{\partial \mu}\mathbb{E}\, u =& -\kappa_1^2(1-\alpha^\star)\frac{2\kappa_1^\star - \kappa_1}{\kappa_1}
\end{aligned}$$

$$\frac{\partial^2}{\partial \mu^2}\mathbb{E}\, u = 2\kappa_1^2(1-\alpha^\star)\left\{\frac{(2p-1)^2(1-\alpha)}{p(1-p)}\left(\frac{2\kappa_1^\star-\kappa_1}{\kappa_1}-\frac{\alpha^\star-\alpha}{1-\alpha^\star}\right)+\frac{2\kappa_1^\star-\kappa_1}{\kappa_1}\right\}$$

Hence utility behaves symmetrically around $\mu = 0$ as around $\mu = 1$. The conditions for the first derivative to be negative are the same as for the derivative to be positive around 1, and conditions for convexity/concavity are the same as equation (Convexity at $\{0, 1\}$).

Finally, let us analyze the case in which $\mu = \frac{1}{2}$:

$$\begin{aligned}
\bar{k} &= \frac{p(1+\alpha-2\alpha p)}{1-\alpha+4\alpha p(1-p)}\\
\bar{k}' &= \frac{4p(1-p)}{1-\alpha+4\alpha p(1-p)}\\
\bar{k}'' &= -\frac{16p(2p-1)(1-\alpha)(1-p)}{\Big(1-\alpha+4\alpha p(1-p)\Big)^2}\\
\underline{k} &= \frac{(1-p)(1-\alpha+2\alpha p)}{1-\alpha+4\alpha p(1-p)}\\
\underline{k}' &= \frac{4p(1-p)}{1-\alpha+4\alpha p(1-p)}\\
\underline{k}'' &= \frac{16p(2p-1)(1-\alpha)(1-p)}{\Big(1-\alpha+4\alpha p(1-p)\Big)^2}\\
\mathbb{E}\, u &= \kappa_1^2 p(1-p)\left(\frac{(1-\alpha^\star)-(2p-1)^2(\alpha^\star+\alpha^2-2\alpha\alpha^\star)}{(4\alpha p^2-4\alpha p+\alpha-1)^2}\right)+\\
&\quad -2\kappa_1\kappa_1^\star p(1-p)\left(\frac{(1-\alpha^\star)-(2p-1)^2\alpha(1-\alpha^\star)}{(4\alpha p^2-4\alpha p+\alpha-1)^2}\right)
\end{aligned}$$

$$\frac{\partial}{\partial \mu}\mathbb{E}\, u = 0$$

$$\frac{\partial^2}{\partial \mu^2}\mathbb{E}\, u = \frac{8p(1-p)\kappa_1^2(1-\alpha^\star)}{\Big(1-\alpha+4\alpha p(1-p)\Big)^2}\left(\frac{2\kappa_1^\star-\kappa_1}{\kappa_1}4p(1-p)+\frac{8\frac{\alpha^\star-\alpha}{1-\alpha^\star}p(1-p)(2p-1)^2(1-\alpha)}{1-\alpha+4\alpha p(1-p)}\right)$$

Hence $\mu = \frac{1}{2}$ is either a (local) maximum or minimum. Moreover, the second derivative is positive if and only if

$$\frac{2\kappa_1^\star-\kappa_1}{\kappa_1} \geq \left(\frac{\alpha-\alpha^\star}{1-\alpha^\star}\right)\left(\frac{2(2p-1)^2(1-\alpha)}{(1-\alpha)+4\alpha p(1-p)}\right)$$

The first term on the RHS vanishes for $\alpha^\star \to \alpha$, the second term vanishes for either $p \to \frac{1}{2}$, or $p \to 1$, or $\alpha \to 1$. The LHS vanishes for $\kappa^\star \to \frac{1}{2}\kappa_1$.

Given the fact that welfare is twice continuously differentiable, when the second derivative is strictly positive, it is weakly positive over a closed interval around $\{0, \frac{1}{2}, 1\}$. □

Lemma 10. *There exist (possibly degenerate) intervals $[0, \mu^\ell)$, (μ^ℓ, μ^h), and $(\mu^h, 1]$ with $0 \leq \mu^\ell < \frac{1}{2} < \mu^h \leq 1$, such that:*

- *For $\mu \in [0, \mu^\ell) \cup (\mu^h, 1]$, welfare is (strictly) convex if and only if*

$$\frac{2\kappa_1^\star - \kappa_1}{\kappa_1} > -\left(\frac{\alpha - \alpha^\star}{1 - \alpha^\star}\right)\left(\frac{(2p-1)^2(1-\alpha)}{(2p-1)^2(1-\alpha) + p(1-p)}\right)$$

- *For $\mu \in (\mu^\ell, \mu^h)$, welfare is (strictly) convex if and only if*

$$\frac{2\kappa_1^\star - \kappa_1}{\kappa_1} > \left(\frac{\alpha - \alpha^\star}{1 - \alpha^\star}\right)\left(\frac{2(2p-1)^2(1-\alpha)}{(1-\alpha) + 4\alpha p(1-p)}\right)$$

Proof. It is immediate to see that $\bar{k}$ and $\underline{k}$ are second-order polynomials in μ. Hence, welfare is a fourth-order polynomial in μ and the second derivative with respect to μ has at most two zeros. In particular, since welfare is symmetric in $[0, 1]$ around $\frac{1}{2}$, it has either two zeros or none in $(0, 1)$.

If the second derivative does not change sign in $(0, 1)$, then checking its sign at $\mu = \frac{1}{2}$ is enough to know the convexity over $(0, 1)$, and the left and right intervals are empty with $\mu^\ell = 0$ and $\mu^h = 1$. If the second derivative changes sign twice, let μ^ℓ and μ^h be the zeros. In that case, since the first derivative is zero at $\mu = \frac{1}{2}$, it cannot be that also the second derivative is zero at $\mu = \frac{1}{2}$, otherwise welfare would not be symmetric. Hence $\mu^\ell < \frac{1}{2} < \mu^h$. The remaining cases are trivial. $\square$

There are only two remaining cases in which the concavification of welfare coincides with the segment that connects welfare at zero and one (i.e. $\mathbb{E}\, u(\mu) \leq 0$ for all μ), and welfare is not convex over the entire domain. The first case is when the first derivative is negative (positive) around $\mu = 0$ ($\mu = 1$) and the second derivative is negative at the extremes. This gives condition (Decreasing at 0). The second case is when the second derivative is positive around $\mu = \frac{1}{2}$ but $\mathbb{E}\, u(\frac{1}{2}) \leq 0$. This gives condition (Negative at 1/2). Since there are at most two zeros of the second derivative and $\mathbb{E}\, u(0) = \mathbb{E}\, u(1) = 0$, there are no other possible cases. Finally, $\mu^\ell = 1 - \mu^h$ as welfare is symmetric around $1/2$.

The rest of the proof follows directly from Kamenica and Gentzkow (2011).

C.10 Proof of corollary 1

When $\kappa = \kappa^\star$, condition (3) holds. Regarding condition (5), notice that given the restrictions on the parameters, $(2p-1)^2 < 1$. Moreover,

$$\frac{2\alpha - \alpha^\star - \alpha^2}{1 - \alpha^\star} < \frac{2\alpha + 1 - \alpha^2}{2}$$

because it is decreasing in $\alpha^\star$. Finally, $\frac{2\alpha + 1 - \alpha^2}{2} < 1$. Hence the RHS of condition (5) is bounded above by the LHS.

C.11 Proof of corollary 2

Using inequalities (Convexity at $\{0,1\}$) and (Convexity at 1/2), it is immediate to find the threshold $\bar{\kappa}_1^\star$:

$$\bar{\kappa}_1^\star(p,\alpha,\kappa_1,\alpha^\star) := \begin{cases} \frac{1}{2}\kappa_1 \left\{1 - \left(\frac{\alpha-\alpha^\star}{1-\alpha^\star}\right)\left(\frac{(2p-1)^2(1-\alpha)}{(2p-1)^2(1-\alpha)+p(1-p)}\right)\right\} & \alpha < \alpha^\star \\ \frac{1}{2}\kappa_1 \left\{1 + \left(\frac{\alpha-\alpha^\star}{1-\alpha^\star}\right)\left(\frac{2(2p-1)^2(1-\alpha)}{(1-\alpha)+4\alpha p(1-p)}\right)\right\} & \alpha \geq \alpha^\star \end{cases}$$

When $\mu^\ell = \underline{\mu}^m$ and $\mu^h = \bar{\mu}^m$, welfare is convex over $(0,1)$. Hence, the optimal policy is fully revealing.

Taking derivatives of the threshold with respect to parameters:

$$\frac{\partial}{\partial p}\bar{\kappa}_1^\star = \begin{cases} \frac{1}{2}\kappa_1 \left(\frac{\alpha^\star-\alpha}{1-\alpha^\star}\right) \frac{(2p-1)(1-\alpha)}{\left((2p-1)^2(1-\alpha)+p(1-p)\right)^2} & \alpha < \alpha^\star \\ \kappa_1 \left(\frac{\alpha-\alpha^\star}{1-\alpha^\star}\right) \frac{4(2p-1)(1-\alpha)}{\left((1-\alpha)+4\alpha p(1-p)\right)^2} & \alpha \geq \alpha^\star \end{cases}$$

This implies $\frac{\partial}{\partial p}\bar{\kappa}_1^\star > 0$.

$$\frac{\partial}{\partial \alpha}\bar{\kappa}_1^\star = \begin{cases} \frac{1}{2}\kappa_1 \left(\frac{(2p-1)^2}{1-\alpha^\star}\right)\left(\frac{[2\alpha-\alpha^\star-1]p(1-p)-(1-\alpha)^2(2p-1)^2}{\left((2p-1)^2(1-\alpha)+p(1-p)\right)^2}\right) & \alpha < \alpha^\star \\ \frac{1}{2}\kappa_1 \left(\frac{2(2p-1)^2}{1-\alpha^\star}\right)\left(\frac{(1-\alpha)^2+(\alpha^\star-\alpha^2)4p(1-p)}{\left((1-\alpha)+4\alpha p(1-p)\right)^2}\right) & \alpha \geq \alpha^\star \end{cases}$$

This implies

$$\frac{\partial}{\partial \alpha}\bar{\kappa}_1^\star = \begin{cases} \geq 0 & \alpha < \alpha^\star \leq (2\alpha-1) - (1-\alpha)^2 \frac{(2p-1)^2}{p(1-p)} \\ \geq 0 & \alpha \geq \alpha^\star \geq \alpha^2 - \frac{(1-\alpha)^2}{4p(1-p)} \\ < 0 & \text{otherwise} \end{cases}$$

$$\frac{\partial}{\partial \kappa_1}\bar{\kappa}_1^\star = \begin{cases} \frac{1}{2} \left\{1 - \left(\frac{\alpha-\alpha^\star}{1-\alpha^\star}\right)\left(\frac{(2p-1)^2(1-\alpha)}{(2p-1)^2(1-\alpha)+p(1-p)}\right)\right\} & \alpha < \alpha^\star \\ \frac{1}{2} \left\{1 + \left(\frac{\alpha-\alpha^\star}{1-\alpha^\star}\right)\left(\frac{2(2p-1)^2(1-\alpha)}{(1-\alpha)+4\alpha p(1-p)}\right)\right\} & \alpha \geq \alpha^\star \end{cases}$$

This implies $\frac{\partial}{\partial \kappa_1}\bar{\kappa}_1^\star > 0$.

Finally,

$$\frac{\partial}{\partial \alpha^\star}\bar{\kappa}_1^\star = \begin{cases} \frac{1}{2}\kappa_1 \left(\frac{1-\alpha}{1-\alpha^\star}\right)^2 \left(\frac{(2p-1)^2}{(2p-1)^2(1-\alpha)+p(1-p)}\right) & \alpha < \alpha^\star \\ \frac{1}{2}\kappa_1 \left(\frac{1-\alpha}{1-\alpha^\star}\right)^2 \left(\frac{2(2p-1)^2}{(1-\alpha)+4\alpha p(1-p)}\right) & \alpha \geq \alpha^\star \end{cases}$$

This implies $\frac{\partial}{\partial \alpha^\star}\bar{\kappa}_1^\star > 0$.

C.12 Proof of corollary 3

Using inequalities (Convexity at $\{0,1\}$) and (Convexity at $1/2$), it is immediate to find the threshold $\bar{\kappa}_1^\star$:

$$\underline{\kappa}_1^\star(p,\alpha,\kappa_1,\alpha^\star) := \begin{cases} \frac{1}{2}\kappa_1 \left\{1 - \left(\frac{\alpha^\star-\alpha}{1-\alpha^\star}\right)\left(\frac{2(2p-1)^2(1-\alpha)}{(1-\alpha)+4\alpha p(1-p)}\right)\right\} & \alpha < \alpha^\star \\ \frac{1}{2}\kappa_1 \left\{1 - \left(\frac{\alpha-\alpha^\star}{1-\alpha^\star}\right)\left(\frac{(2p-1)^2(1-\alpha)}{(2p-1)^2(1-\alpha)+p(1-p)}\right)\right\} & \alpha \geq \alpha^\star \end{cases}$$

When $\mu^\ell = \underline{\mu}^m$ and $\mu^h = \bar{\mu}^m$, welfare is concave over $(0,1)$. Hence, it is immediate from Kamenica and Gentzkow (2011) that the optimal policy is revealing. The rest of the proof follows the same lines as in corollary (2).

Taking derivatives of the threshold with respect to parameters:

$$\frac{\partial}{\partial p}\underline{\kappa}_1^\star = \begin{cases} -\kappa_1\left(\frac{\alpha^\star-\alpha}{1-\alpha^\star}\right)\frac{4(2p-1)(1-\alpha)}{\left((1-\alpha)+4\alpha p(1-p)\right)^2} & \alpha < \alpha^\star \\ -\frac{1}{2}\kappa_1\left(\frac{\alpha-\alpha^\star}{1-\alpha^\star}\right)\frac{(2p-1)(1-\alpha)}{\left((2p-1)^2(1-\alpha)+p(1-p)\right)^2} & \alpha \geq \alpha^\star \end{cases}$$

This implies $\frac{\partial}{\partial p}\underline{\kappa}_1^\star < 0$.

$$\frac{\partial}{\partial \alpha}\underline{\kappa}_1^\star = \begin{cases} \frac{1}{2}\kappa_1\left(\frac{2(2p-1)^2}{1-\alpha^\star}\right)\left(\frac{(1-\alpha)^2+(\alpha^\star-\alpha^2)4p(1-p)}{\left((1-\alpha)+4\alpha p(1-p)\right)^2}\right) & \alpha < \alpha^\star \\ \frac{1}{2}\kappa_1\left(\frac{(2p-1)^2}{1-\alpha^\star}\right)\left(\frac{[2\alpha-\alpha^\star-1]p(1-p)-(1-\alpha)^2(2p-1)^2}{\left((2p-1)^2(1-\alpha)+p(1-p)\right)^2}\right) & \alpha \geq \alpha^\star \end{cases}$$

This implies

$$\frac{\partial}{\partial \alpha}\underline{\kappa}_1^\star = \begin{cases} \leq 0 & \alpha < \alpha^\star \leq \alpha^2 - \frac{(1-\alpha)^2}{4p(1-p)} \\ \leq 0 & \alpha \geq \alpha^\star \geq (2\alpha-1)-(1-\alpha)^2\frac{(2p-1)^2}{p(1-p)} \\ > 0 & \text{otherwise} \end{cases}$$

$$\frac{\partial}{\partial \kappa_1}\underline{\kappa}_1^\star = \begin{cases} \frac{1}{2}\left\{1 - \left(\frac{\alpha^\star-\alpha}{1-\alpha^\star}\right)\left(\frac{2(2p-1)^2(1-\alpha)}{(1-\alpha)+4\alpha p(1-p)}\right)\right\} & \alpha < \alpha^\star \\ \frac{1}{2}\left\{1 - \left(\frac{\alpha-\alpha^\star}{1-\alpha^\star}\right)\left(\frac{(2p-1)^2(1-\alpha)}{(2p-1)^2(1-\alpha)+p(1-p)}\right)\right\} & \alpha \geq \alpha^\star \end{cases}$$

This implies that $\frac{\partial}{\partial \kappa_1} > 0$ when $\alpha \geq \alpha^\star$ and ambiguous otherwise, but always positive for $\alpha^\star - \alpha$ small enough.

Finally,

$$\frac{\partial}{\partial \alpha^\star}\underline{\kappa}_1^\star = \begin{cases} \frac{1}{2}\kappa_1\left(\frac{1-\alpha}{1-\alpha^\star}\right)^2\left(\frac{2(2p-1)^2}{(1-\alpha)+4\alpha p(1-p)}\right) & \alpha < \alpha^\star \\ \frac{1}{2}\kappa_1\left(\frac{1-\alpha}{1-\alpha^\star}\right)^2\left(\frac{(2p-1)^2}{(2p-1)^2(1-\alpha)+p(1-p)}\right) & \alpha \geq \alpha^\star \end{cases}$$

This implies $\frac{\partial}{\partial \alpha^\star}\underline{\kappa}_1^\star > 0$.

C.13 Proof of corollaries 4 - 5

For corollary (4), rearranging conditions (Convexity at $\{0,1\}$)-(Convexity at 1/2) leads to

$$\frac{\alpha-\alpha^\star}{1-\alpha^\star} \geq \begin{cases} \frac{2\kappa_1^\star-\kappa_1}{\kappa_1}\left(\frac{(1-\alpha)+4\alpha p(1-p)}{2(2p-1)^2(1-\alpha)}\right) & \kappa_1^\star \geq \frac{1}{2}\kappa_1 \\ \frac{\kappa_1-2\kappa_1^\star}{\kappa_1}\left(\frac{(2p-1)^2(1-\alpha)+p(1-p)}{(2p-1)^2(1-\alpha)}\right) & \kappa_1^\star < \frac{1}{2}\kappa_1 \end{cases}$$

implies that welfare is convex over $[0,\mu^\ell]\cup[\mu^h,1]$ and concave over $[\underline{\mu}^m,\bar{\mu}^m]$. This implies the form of the optimal policy. Finally, τ can be found using the Bayes-plausibility constraint and π^y using formula (11).

For corollary (5),

$$\frac{\alpha-\alpha^\star}{1-\alpha^\star} \leq \begin{cases} \frac{2\kappa_1^\star-\kappa_1}{\kappa_1}\left(\frac{(1-\alpha)+4\alpha p(1-p)}{2(2p-1)^2(1-\alpha)}\right) & \kappa_1^\star \geq \frac{1}{2}\kappa_1 \\ \frac{\kappa_1-2\kappa_1^\star}{\kappa_1}\left(\frac{(2p-1)^2(1-\alpha)+p(1-p)}{(2p-1)^2(1-\alpha)}\right) & \kappa_1^\star < \frac{1}{2}\kappa_1 \end{cases}$$

implies that welfare is concave over $[0,\mu^\ell]\cup[\mu^h,1]$ and convex over $[\underline{\mu}^m,\bar{\mu}^m]$. This implies the form of the optimal policy.

D Illustration: Model with Real Rigidities

As a second illustration for the paper, the business-cycle models of Angeletos et al. (2016) are modified and extended to fit exactly the framework studied here. The main point of this example is to show that also real rigidities can be accommodated by the theoretical framework.

D.1 A Stylized Model of Central Bank Communication with Real Rigidities

The economy consists of a "mainland" and a continuum of "islands." Each island is inhabited by a continuum of workers and a continuum of monopolistically competitive firms. Inputs produced in islands are aggregated in intermediate goods, intermediate goods are aggregated in a final good. The final good is sold in the mainland and consumed by a representative household, which collects all the labor income from local workers and owns all the firms.

Household. The representative household enjoys utility from consumption and disutility from working. Utility of the representative household is given by

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t \left[U(C_t) + \int_I \int_J V(n_{ijt}) dj di \right]$$

where $U(C) \equiv \frac{1}{1-\gamma} C^{1-\gamma}$ and[46] $V(n) \equiv -\frac{1}{1+\epsilon} n^{1+\epsilon}$, n_{ijt} is labor input in firm j in island i and C_t is consumption of the final good.

The final good C_t is aggregated using a CES technology from island-specific intermediate goods c_{it}, where i denotes an island. Intermediate goods are obtained by aggregating inputs c_{ijt}, produced locally by firm j in island i. I assume a standard nested CES structure with elasticity of substitution ρ across islands and η_t within islands.

$$C_t = \left[\int_I c_{it}^{\frac{\rho-1}{\rho}} di \right]^{\frac{\rho}{\rho-1}}, \quad c_{it} = \left[\int_J c_{ijt}^{\frac{\eta_t-1}{\eta_t}} dj \right]^{\frac{\eta_t}{\eta_t-1}}$$

where $\rho > \eta_t > 1$ for all t. The elasticity within islands is allowed to change over time, and this generates countercyclical fluctuations in the aggregate markup, common across islands. The household maximizes utility by choosing labor supply and consumption subject to the aggregate budget constraint

$$\int_I \int_J p_{ijt} c_{ijt} dj di \leq \int_I w_{it} n_{it} di + \int_I \int_J \pi_{ijt} dj di$$

where w_{it} is the nominal wage in island i, π_{ijt} are profits of of the firm j in island i, and p_{ijt} is the price of input i from island j.

[46] Assume that ϵ is not too large such that $\alpha > -1$. The assumption is not restrictive for usual calibrations.

Final Good Producer. The final good producer has a production technology $Y_t^{1-\omega}$, where the parameter ω can accommodate increasing, constant, or decreasing returns to scale. The parameter is a simple way of parameterizing inefficient coordination levels among firms, arising for example from Cournot or Bertrand competition. With the normalization $P_t = 1$, profits of the final good producer are thus

$$Y_t^{1-\omega} - \int p_{it} y_{it} di$$

which are maximized by choosing inputs y_{it} subject to the CES aggregator $Y_t = \left[\int_I y_{it}^{\frac{\rho-1}{\rho}} di\right]^{\frac{\rho}{\rho-1}}$. This leads to the downward-sloping demand curve[47] for intermediate good y_{it}

$$y_{it} = \left(\frac{p_{it}}{(1-\omega)}\right)^{-\rho} Y_t^{1-\rho\omega}$$

Intermediate Good Producers. Intermediate good producers purchase inputs from within each island and produces an island specific good y_{it}. it follows from the nested CES structure that

$$y_{ijt} = \left(\frac{p_{ijt}}{p_{it}}\right)^{-\eta_t} y_{it}$$

and the island price index is for each island $i \in I$

$$p_{it}^{1-\eta_t} = \int_J p_{ijt}^{1-\eta_t} dj$$

Firms. Firms in islands employ local workers through a competitive labor market and produce differentiated inputs using a constant return technology, with labor as the only input. Every island is identical in terms of production technology, but differs in terms of the information received about the state of the economy. Firms choose production to maximize expected discounted profits, based on imperfect information about the aggregate demand. Information within an island is shared to all the firms, so that production within islands is homogeneous and differences across islands are only driven by heterogeneity in the information received. Prices are set at the level of the firm flexibly after uncertainty is resolved, so that markets clear.[48]

The production function of firm i in island j is $y_{ijt} = A_t n_{ijt}$, where A_t is an aggregate productivity, and profits are given by $\Pi_{ijt} = p_{ijt} y_{ijt} - w_{it} n_{ijt}$. The

[47] Notice that even if $\frac{\partial \log y_{it}}{\partial \log Y_t} > 0$ (commonly referred in macroeconomics as "strategic complementarities"), this economy features strategic substitutes because $\frac{\partial^2 \mathcal{U}_t}{\partial \log y_{it} \partial \log Y_t} < 0$.

[48] Extending the model to include price stickiness would be easy along the lines of Angeletos et al. (2016) and Angeletos and La'O (2020), but one would need to introduce a second production input for markets to clear at the cost of more complications. See the application with only nominal rigidities for more insights.

firm maximizes expected discounted real profits conditional on the information available and subject to the demand curve for the input $p_{ijt} = D_{it} y_{ijt}^{-\frac{1}{\eta_t}}$, where

$$D_{it} = p_{it} y_{it}^{\frac{1}{\eta_t}} = (1-\omega) Y_t^{\frac{1}{\rho}-\omega} y_{it}^{\frac{1}{\eta_t}-\frac{1}{\rho}}$$

This leads to

$$\max_{y_{ijt}} \mathbb{E}_{it} \left\{ Y_t^{-\gamma} \left(A_t D_{it} y_{ijt}^{1-\frac{1}{\eta_t}} - w_{it} y_{ijt} \right) \right\}$$

Business-cycle fluctuations. Following Angeletos et al. (2016), I will focus on two sources of aggregate fluctuations. The first is the elasticity of substitution between inputs $\eta(\theta_t)$, that is allowed to vary depending on the state of the economy θ_t, which is a binary i.i.d. random variable. In other words, the first source of fluctuations in this economy are (countercyclical) markup shocks, with $\log \mathcal{M}(\theta_t) \equiv \log \left(\frac{\eta(\theta_t)}{\eta(\theta_t)-1} \right) = -(\kappa_0^{\mathcal{M}} + \kappa_1^{\mathcal{M}} \theta_t)$, $\kappa_0^{\mathcal{M}}, \kappa_1^{\mathcal{M}} > 0$. The second source of fluctuations is an aggregate technology shock, with $\log A(\theta_t) = \kappa_0^A + \kappa_1^A \theta_t$, $\kappa_0^A, \kappa_1^A > 0$. I will denote with a subscript t the dependence on θ_t to ease the notation.

Information. Information is the same as in the abstract setting. Agents observe a private signal x_{it} informative about θ_t, correct with probability p, and a public signal z_t, chosen optimally with commitment by the central bank to maximize utility of the household. Information about θ_t available to the monetary authority is incomplete. This implies that the public signal is chosen optimally subject to an information aggregation constraint, which captures the ability of the planner to aggregate information that is dispersed across islands.

Mapping to the Abstract Setting. The next propositions clarify in which sense the economy introduced is an illustration for the abstract model discussed.

Proposition 9 (Best Response)**.** *Let $\tilde{Y}_t$ be the unique production level in the islands economy under complete information. Up to a first-order approximation around $\log \tilde{Y}_t$, the optimal production y_{ijt} of firm j in island i is given by*

$$\log y_{ijt} = \log y_{it} = (1-\alpha)\, \mathbb{E}_{it} \log \tilde{Y}_t + \alpha\, \mathbb{E}_{it} \log Y_t$$

where $\alpha := \frac{1-\rho(\gamma+\omega)}{1+\rho\epsilon}$, $Y_t = \left(\int_I y_{it}^{\frac{\rho-1}{\rho}} di \right)^{\frac{\rho}{\rho-1}}$, and $\mathbb{E}_{it}$ denotes the expectation conditional on information available to island i at time t.

Proposition (9) is the equivalent of equation (Best response). Individual actions in the decentralized equilibrium are a linear combination of conditional expectations of the frictionless level of aggregate output and actual aggregate demand. The frictionless output can be thought as the natural level of output, where frictions that prevent quantities to adjust flexibly are removed. The weight α, which is the optimal level of coordination in the decentralized equilibrium, is pinned down by preference parameters and production technology of the final good producer. Therefore, with appropriate relabelling, the characterization in proposition (6) is still valid up to a first-order approximation.

When $\omega \neq 0$, the economy features a coordination externality. This comes from the fact that pricing power of the final good producer generates a wedge between the demand curve faced by firms and that implied by household's preferences only (with $\omega = 0$). Moreover, when $\log \mathcal{M} \neq 1$, the economy is inefficient under complete information. On the other hand, fluctuations driven by productivity are efficient. Therefore, this economy potentially features both distortions that have been studied in the abstract setting.

The central bank maximizes expected utility of the representative household by choosing optimally the public communication. With price stickiness, this would lead to a familiar representation of the objective function of the monetary authority in terms of squared output gap, squared inflation, and an inflation bias that is present when fiscal policy does not offset market power with a subsidy.

Proposition 10 (Welfare)**.** *Let $Y_t^\star$ be the efficient level of production that maximizes welfare under complete information. In a second-order approximation around the complete-information benchmark, ex-ante welfare is given by*

$$\mathbb{E}_t\,\mathcal{U}_t = (1-\alpha^\star)\left\{2\mathrm{Cov}_t\left[\log\left(\frac{Y_t}{\tilde{Y}_t}\right), \log\left(\frac{Y_t^\star}{\tilde{Y}_t}\right)\right] - \mathbb{E}_t[(\log Y_t - \log \tilde{Y}_t)^2]\right\} - \mathbb{E}_t\,\sigma^2_{\log y_i}$$

up to a positive multiplicative constant and an additive constant that does not depend on information, with $\alpha^\star := \frac{1-\rho\gamma}{1+\rho\epsilon}$.

The proposition shows a decomposition of welfare in usual three terms, as in lemma (4). The dispersion term $\mathbb{E}_t\,\sigma^2_{\log y_i}$ captures output losses generated from heterogeneity in information. Through the downward sloping demand curve, it can be rewritten in terms of dispersion in prices. In models with price stickiness, this usually is written as inflation squared. The volatility term $\mathbb{E}_t[(\log Y_t - \log \tilde{Y}_t)^2]$ is the volatility of the output gap, where the natural level of output is the output that realizes without information frictions. The covariance term $\mathrm{Cov}\left[\log\left(\frac{Y_t}{\tilde{Y}_t}\right), \log\left(\frac{Y_t^\star}{\tilde{Y}_t}\right)\right]$ captures the extent to which fluctuations in the output gap $\log Y_t - \log \tilde{Y}_t$ move the economy closer or farer to the efficiency benchmark $Y_t^\star$. With technology shocks it is always zero. With markup shocks the covariance is positive (if and only if $\mathrm{Corr}(\mathcal{M}_t, \theta_t) = -\kappa_1^{\mathcal{M}} < 0$, that is markups are countercyclical), which means that fluctuations in markups can partially alleviate welfare losses due to volatility and dispersion, because the economy gets closer to efficiency. Through a public communication, the central bank can affect welfare via these three channels, and sometimes even substitute for missing policy instruments.

D.2 Proofs for Appendix D

D.2.1 Proof of proposition 9

The proof is for $\log \mathcal{M}_t = -\kappa_0^{\mathcal{M}} - \kappa_1^{\mathcal{M}}\theta_t$ and $A_t = 1$. Similar steps would show that the result holds for $\mathcal{M}_t = 1$ and $\log A_t = \kappa_0^A + \kappa_1^A\theta_t$. Firms' first order condition is

$$\mathbb{E}_{it}\left\{\frac{Y_t^{-\gamma}}{P_t}\left[\left(1-\frac{1}{\eta_t}\right)D_{it}y_{ijt}^{-\frac{1}{\eta_t}} - w_{it}\right]\right\} = 0$$

Using the fact that, within each island, firms are symmetric, we have that $y_{ijt} = y_{it}$ and $n_{ijt} = n_{it}$. This leads to

$$\mathbb{E}_{it}\left\{\left(1-\frac{1}{\eta_t}\right)(1-\omega)Y_t^{\frac{1}{\rho}-\omega-\gamma}y_{it}^{-\frac{1}{\rho}} - \frac{Y_t^{-\gamma}}{P_t}w_{it}\right\} = 0$$

Using $P_tC_t = \int_I\int_J p_{ijt}c_{ijt}djdi$ in the household budget constraint and taking derivatives, the optimal labor supply of the household is

$$n_{ijt}^{\epsilon} = n_{it}^{\epsilon} = \frac{w_{it}}{P_t}Y_t^{-\gamma}$$

Replacing in the FOC for firms

$$\mathbb{E}_{it}\left\{\left(1-\frac{1}{\eta_t}\right)(1-\omega)Y_t^{\frac{1}{\rho}-\omega-\gamma}y_{it}^{-\frac{1}{\rho}} - y_{it}^{\epsilon}\right\} = 0$$

$$y_{it} = \left(\mathbb{E}_{it}\left\{\frac{1}{\mathcal{M}_t}(1-\omega)Y_t^{\frac{1-\rho(\gamma+\omega)}{\rho}}\right\}\right)^{\frac{\rho}{1+\rho\epsilon}} \tag{23}$$

Let us now derive the complete-information allocation, which will be denoted with a tilde. If information is complete, one can drop the expectation and solve for $\tilde{y}_{it}$ and $\tilde{Y}_t$. Since the only source of heterogeneity is the information set, firms will produce the same quantities and charge the same price, that is

$$\tilde{y}_{it} = \tilde{n}_{it} = \tilde{Y}_t = \left(\frac{1-\omega}{\mathcal{M}_t}\right)^{\frac{1}{\gamma+\epsilon+\omega}}$$
$$\tilde{p}_{it} = P_t\frac{1}{\mathcal{M}_t^{FP}}$$
$$\tilde{w}_{it} = \tilde{w}_t = P_t\frac{1}{\mathcal{M}_t}$$

Using the binary state, this implies $\log\tilde{y}(0) = \log\tilde{Y}(0) = \frac{1}{\gamma+\epsilon+\omega}(\log(1-\omega)+\kappa_0^{\mathcal{M}})$ and $\log\tilde{y}(1) = \log\tilde{Y}(1) = \frac{1}{\gamma+\epsilon+\omega}(\log(1-\omega)+\kappa_0^{\mathcal{M}}+\kappa_1^{\mathcal{M}})$.

Log-linearizing equation (23) around the complete-information allocation at $\theta_t = 0$:

$$
\begin{aligned}
y_{it}^{\frac{1+\rho\epsilon}{\rho}} &= \mathbb{E}_{it}\left[\exp\left(\log(1-\omega) - \log\mathcal{M}_t + \frac{1-\rho(\gamma+\omega)}{\rho}\log Y_t\right)\right] \\
&\approx \mathbb{E}_{it}\left[\exp\left(\frac{1+\rho\epsilon}{\rho(\gamma+\epsilon+\omega)}(\log(1-\omega)+\kappa_0^{\mathcal{M}})\right)\cdot\right. \\
&\qquad \left.\cdot\left(1 - (\log\mathcal{M}_t - \kappa_0^{\mathcal{M}}) + \frac{1-\rho(\gamma+\omega)}{\rho}(\log Y_t - \log\tilde{Y}(0))\right)\right]
\end{aligned}
$$

Hence, up to a first order approximation

$$
\begin{aligned}
\log y_{it} - \log\tilde{y}(0) &= -\frac{\rho}{1+\rho\epsilon}\,\mathbb{E}_{it}(\log\mathcal{M}_t - \kappa_0^{\mathcal{M}}) + \frac{1-\rho(\gamma+\omega)}{1+\rho\epsilon}\,\mathbb{E}_{it}(\log Y_t - \log\tilde{Y}(0)) \\
&= \frac{\rho}{1+\rho\epsilon}\kappa_1^{\mathcal{M}}\,\mathbb{E}_{it}(\theta_t) + \frac{1-\rho(\gamma+\omega)}{1+\rho\epsilon}\,\mathbb{E}_{it}(\log Y_t - \log\tilde{Y}(0)) \\
&= (1-\alpha)\kappa_1\,\mathbb{E}_{it}(\theta_t) + \alpha\,\mathbb{E}_{it}(\log Y_t - \log\tilde{Y}(0))
\end{aligned}
$$

where $\alpha := \frac{1-\rho(\gamma+\omega)}{1+\rho\epsilon}$ and $\kappa_1 := \frac{\kappa_1^{\mathcal{M}}}{\gamma+\epsilon+\omega}$. Rearranging and using the fact that $\tilde{y}_{it} = \tilde{Y}_t$,

$$
\log y_{it} = (1-\alpha)\,\mathbb{E}_{it}\log\tilde{Y}_t + \alpha\,\mathbb{E}_{it}\log Y_t
$$

where $\log\tilde{Y}(\theta_t) = -\frac{1}{\gamma+\epsilon+\omega}(\log\mathcal{M}_t) = \kappa_0 + \kappa_1\theta_t$, and $\kappa_0 := \frac{\kappa_0^{\mathcal{M}}}{\gamma+\epsilon+\omega}$.

D.2.2 Proof of proposition 10

The per-period utility is

$$
\mathcal{U}_t = \frac{1}{1-\gamma}C_t^{1-\gamma} - \int_I \frac{1}{1+\epsilon}n_{it}^{1+\epsilon}di
$$

Denote with $\hat{x}_{it} := \log x_{it} - \log\tilde{x}_t$ the log deviations from the complete-information allocation. A second-order expansion of $\mathcal{U}_t$ around $\tilde{\mathcal{U}}$ yields

$$
\begin{aligned}
\mathcal{U}_t - \tilde{\mathcal{U}} &\approx \tilde{U}'\tilde{Y}\left(\hat{Y}_t + \frac{1}{2}\hat{Y}_t^2\right) + \tilde{V}'\tilde{n}_i\int_I\left(\hat{n}_{it} + \frac{1}{2}\hat{n}_{it}^2\right)di + \frac{1}{2}\tilde{U}''\tilde{Y}^2\hat{Y}_t^2 + \frac{1}{2}\tilde{V}''\tilde{n}_i^2\int_I\hat{n}_{it}^2 di \\
&= \tilde{Y}^{1-\gamma}\left(\hat{Y}_t + \frac{1-\gamma}{2}\hat{Y}_t^2\right) - \tilde{n}_{it}^{1+\epsilon}\int_I\left(\hat{y}_{it} + \frac{1+\epsilon}{2}\hat{y}_{it}^2\right)di
\end{aligned}
$$

where the market clearing conditions has been used to substitute for C_t and the production function to substitute for n_{it}. Taking expectations over θ_t and using $\tilde{n}_{it} = \tilde{Y}_t$

$$
\mathbb{E}_t\left\{\frac{\mathcal{U}_t - \tilde{\mathcal{U}}}{\tilde{Y}^{1-\gamma}}\right\} \approx \mathbb{E}_t\left(\hat{Y}_t + \frac{1-\gamma}{2}\hat{Y}_t^2\right) - \mathbb{E}_t\left(\tilde{n}_{it}^{\epsilon}\tilde{Y}_t^{\gamma}\int_I\left(\hat{y}_{it} + \frac{1+\epsilon}{2}\hat{y}_{it}^2\right)di\right)
$$

Replacing the labor supply and the wage in the complete-information allocation $\tilde{w}_t = P_t \frac{1}{\mathcal{M}_t}$

$$\mathbb{E}_t \left\{ \frac{\mathcal{U}_t - \tilde{\mathcal{U}}}{\tilde{Y}^{1-\gamma}} \right\} \approx \mathbb{E}_t \left(\hat{Y}_t + \frac{1-\gamma}{2} \hat{Y}_t^2 \right) - \mathbb{E}_t \left(\frac{1}{\mathcal{M}_t} \int_I \left(\hat{y}_{it} + \frac{1+\epsilon}{2} \hat{y}_{it}^2 \right) di \right)$$

Adding and subtracting $\mathbb{E}_t \frac{1}{\mathcal{M}_t} \log Y_t$

$$\begin{aligned} \mathbb{E}_t \left\{ \frac{\mathcal{U}_t - \tilde{\mathcal{U}}}{\tilde{Y}^{1-\gamma}} \right\} \approx & \, \mathbb{E}_t \left[\left(1 - \frac{1}{\mathcal{M}_t} \right) \hat{Y}_t + \frac{1-\gamma}{2} \hat{Y}_t^2 \right] \\ & - \mathbb{E}_t \left[\frac{1}{\mathcal{M}_t} \left(\int_I (\log y_{it} - \log Y_t) di + \frac{1+\epsilon}{2} \int_I \hat{y}_{it}^2 di \right) \right] \end{aligned}$$

Now, using $\log \tilde{y}_{it} = \log \tilde{Y}_t$, take a second-order expansion of

$$\left(\frac{y_{it}}{Y_t} \right)^{\frac{\rho-1}{\rho}} \approx 1 + \frac{\rho-1}{\rho} (\log y_{it} - \log Y_t) + \frac{1}{2} \left(\frac{\rho-1}{\rho} \right)^2 (\log y_{it} - \log Y_t)^2$$

Using the definition of the aggregator $Y_t^{\frac{\rho-1}{\rho}} = \int_I y_{it}^{\frac{\rho-1}{\rho}} di$,

$$\int_I (\log y_{it} - \log Y_t) di \approx -\frac{1}{2} \frac{\rho-1}{\rho} \int_I (\log y_{it} - \log Y_t)^2 di = -\frac{1}{2} \frac{\rho-1}{\rho} \sigma^2_{\log y_i}$$

Adding and subtracting $\log Y_t$, the term $\int_I \hat{y}_{it}^2 di$ can be rewritten as

$$\begin{aligned} \int_I \left(\log y_{it} - \log \tilde{Y}_t \right)^2 di = & \sigma^2_{\log y_i} + \left(\log Y_t - \log \tilde{Y}_t \right)^2 \\ & + 2(\log Y_t - \log \tilde{Y}_t) \left(\int_I (\log y_{it} - \log Y_t) di \right) \\ \approx & \sigma^2_{\log y_i} + \left(\log Y_t - \log \tilde{Y}_t \right)^2 \end{aligned}$$

where the last term is negligible up to second order. Substituting back

$$\begin{aligned} \mathbb{E}_t \left\{ \frac{\mathcal{U}_t - \tilde{\mathcal{U}}}{\tilde{Y}^{1-\gamma}} \right\} \approx & \, \mathbb{E}_t \left[\left(1 - \frac{1}{\mathcal{M}_t} \right) \hat{Y}_t + \frac{1-\gamma}{2} \hat{Y}_t^2 \right] \\ & - \mathbb{E}_t \left[\frac{1}{\mathcal{M}_t} \left(-\frac{1}{2} \frac{\rho-1}{\rho} \sigma^2_{\log y_i} + \frac{1+\epsilon}{2} (\sigma^2_{\log y_i} + \hat{Y}_t) \right) \right] \end{aligned}$$

If the economy is efficient ($\mathcal{M}_t = 1$ for all t), Then the above simplifies to[49]

$$\mathbb{E}_t \left\{ \frac{\mathcal{U}_t - \tilde{\mathcal{U}}}{\tilde{Y}^{1-\gamma}} \right\} \approx -\frac{1}{2} \left[(\epsilon + \gamma) \, \mathbb{E}_t \, \hat{Y}_t^2 + \frac{1+\epsilon\rho}{\rho} \, \mathbb{E}_t \, \sigma^2_{\log y_i} \right]$$

[49] Together with $\kappa_1 = \frac{-U_{k\theta}}{U_{kk}+U_{kK}} = \frac{-\kappa_1^{\mathcal{M}}}{\gamma+\epsilon}$, this shows that $(\epsilon+\gamma) = W_{KK}$ and $\frac{1+\epsilon\rho}{\rho} = W_{\sigma\sigma}$. Recall that $(1-\alpha^\star) = \frac{W_{KK}}{W_{\sigma\sigma}} = \frac{U_{kk}+2U_{kK}+U_{KK}}{U_{kk}+U_{\sigma\sigma}}$ here coincides with $(1-\alpha) = \frac{U_{kk}+U_{kK}}{U_{kk}}$. Since $U_{\sigma\sigma} = 0$, here it must be that $U_{kk} = \frac{1+\epsilon\rho}{\rho}$ and $U_{kK} = -U_{KK} = -\frac{1-\rho\gamma}{\rho} < 0$, which shows that the economy features strategic substitutes. Moreover, since $\kappa_1^\star = -\frac{W_{K\theta}}{W_{KK}} = -\frac{U_{k\theta}+U_{K\theta}}{\epsilon+\gamma} = 0$, this implies that $\kappa_1 = -U_{k\theta} = U_{K\theta}$.

Rearranging

$$\mathbb{E}_t \left\{ \frac{\rho(\mathcal{U}_t - \tilde{\mathcal{U}})}{\tilde{Y}^{1-\gamma}(1+\epsilon\rho)} \right\} \approx -\frac{1}{2} \left\{ \frac{\rho(\epsilon+\gamma)}{1+\epsilon\rho} \mathbb{E}_t[(\log Y_t - \log \tilde{Y}_t)^2] + \mathbb{E}_t\, \sigma^2_{\log y_i} \right\}$$
$$= -\frac{1}{2} \left\{ (1-\alpha^\star)\, \mathbb{E}_t[(\log Y_t - \log \tilde{Y}_t)^2] + \mathbb{E}_t\, \sigma^2_{\log y_i} \right\}$$

where $\alpha^\star := \frac{1-\rho\gamma}{1+\epsilon\rho}$ is the efficient level of coordination. Notice that $\alpha^\star = \alpha$ if and only if $\omega = 0$, $\alpha^\star > \alpha$ for $\omega > 0$, and $\alpha^\star < \alpha$ for $\omega < 0$.

If the economy is inefficient, the first-order term does not vanish as $\mathcal{M}_t \neq \mathcal{M}_t^\star \equiv 1$. Under the standard "small distortion" assumption, that is when the distortion has the same order of magnitude as fluctuations in non-fundamental volatility or dispersion, the product of $\frac{1}{\mathcal{M}_t}$ and second-order term can be ignored as negligible. Define $\log Y_t^\star := -\frac{1}{\epsilon+\gamma}(\log \mathcal{M}_t^\star) \equiv \kappa_0^\star + \kappa_1^\star \theta_t$ (symmetrically to $\log \tilde{Y}_t$). Expanding $1 - \frac{1}{\mathcal{M}_t}$, and using the fact that $\mathbb{E}_t\, \hat{Y}_t = 0$,

$$\mathbb{E}_t \left[\left(1 - \frac{1}{\mathcal{M}_t}\right) \hat{Y}_t \right] \approx (\epsilon+\gamma)\mathrm{Cov}_t \left[\log Y_t - \log \tilde{Y}_t, \log Y_t^\star - \log \tilde{Y}_t \right]$$

Putting everything together, this gives

$$\mathbb{E}_t\, \mathcal{U}_t \approx (1-\alpha^\star) \left\{ 2\mathrm{Cov}_t \left[\log\left(\frac{Y_t}{\tilde{Y}_t}\right), \log\left(\frac{Y_t^\star}{\tilde{Y}_t}\right) \right] - \mathbb{E}_t[(\log Y_t - \log \tilde{Y}_t)^2] \right\} - \mathbb{E}_t\, \sigma^2_{\log y_i}$$

up to a positive multiplicative constant and a additive constant across all Bayes-plausible distributions.

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